

# Detection of Surface Submesoscale Fronts in the Atlantic Ocean

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## Abstract

Submesoscale fronts are narrow zones of sharp tracer gradients. They play a significant role in upper-ocean dynamics, vertical exchanges, and biogeochemical processes. However, detecting submesoscale fronts across ocean basins is challenging due to observational limitations and the complexity of frontogenesis processes. This study examines tracer- and velocity-based diagnostics for detecting surface submesoscale fronts using a submesoscale-permitting Atlantic simulation. The Local Variability Index (LVI) identifies thermal, haline, and density fronts, and the Finite-Time Lyapunov Exponent (FTLE) identifies dynamical fronts. All types of fronts have stronger strain, vorticity, convergence, and frontogenesis than the background, with dynamical and density fronts showing the strongest signatures. Dynamical fronts are the most spatially widespread, whereas the distribution of density fronts reflects the combined influence of deformation-driven frontogenesis and thermohaline compensation or reinforcement. Density fronts often co-occur with dynamical and thermohaline fronts, consistent with deformation-driven frontogenesis. FTLE identifies density fronts more effectively than temperature-based LVI in salinity-dominated regions and more effectively than salinity-based LVI in temperature-dominated regions. However, FTLE may also capture elongated deformation features that are spatially offset from instantaneous density fronts, especially where compensation weakens buoyancy gradients. Combining FTLE and LVI enables a more comprehensive detection of density fronts across diverse dynamical regions. FTLE remains effective at mesoscale resolving resolution (~12.5 km) but decreases noticeably at altimetry-like effective resolution (~100 km). LVI requires high-resolution satellite-based data to capture submesoscale variability. This work provides new insights into spatial and seasonal variability of submesoscale fronts and benchmarks front-detection methods in high-resolution simulations and observations.

*Keywords:* Submesoscale, Front, Finite-time Lyapunov exponent, F1-score, Thermohaline compensation, Atlantic

## 36 1 Introduction

37 Submesoscale fronts are narrow zones marked by sharp horizontal gradients in temperature,  
38 salinity, density, or other tracers, and typically exhibit spatial scales of 1–10 km and a lifespan of  
39 1–10 days (Lévy et al., 2023; McWilliams, 2016, 2019; Taylor & Thompson, 2023). They are  
40 characterized by  $O(1)$  Rossby and Richardson numbers (Balwada et al., 2021; Barkan et al., 2019;  
41 Thomas et al., 2008). The departures from geostrophic balance allow strong vertical velocities to  
42 develop. By modulating vertical and lateral fluxes of momentum, heat, and tracers, submesoscale  
43 fronts play a central role in upper-ocean stratification, mixing, biological productivity, air-sea  
44 interactions, and the energy cascade from larger to smaller scales (Freilich et al., 2022; Lévy et al.,  
45 2018; Mahadevan, 2016; Strobach et al., 2022; Zhang et al., 2023). Their ubiquity and dynamical  
46 influence in the upper-ocean make them a key target for advancing ocean modeling and  
47 understanding fine-scale ocean processes.

48 The generation and evolution of submesoscale fronts involve a variety of processes, among  
49 which frontogenesis typically plays a leading role in sharpening and maintaining frontal structures.  
50 Frontogenesis disrupts the geostrophic balance at fronts, inducing an ageostrophic secondary  
51 circulation (Mahadevan et al., 2020; McWilliams, 2021; Taylor & Thompson, 2023). This  
52 generates vertical velocities that are an order of magnitude larger than those driven by mesoscale  
53 eddies, reaching up to 10–100 m day<sup>-1</sup> (Gula et al., 2022; Klein & Lapeyre, 2009). In addition to  
54 frontogenesis, submesoscale fronts are influenced by various other mechanisms, such as mixed-  
55 layer baroclinic instability, symmetric instability, ageostrophic anticyclonic instability, and  
56 horizontal shear instability (Boccaletti et al., 2007; D’Asaro et al., 2011; Gula et al., 2022;  
57 McWilliams et al., 2004; Munk et al., 2000; Thomas & Taylor, 2010). Commonly, these processes  
58 occur simultaneously and their relative contributions to frontal evolution vary geographically and  
59 seasonally (Callies et al., 2015; Rocha et al., 2016; Sasaki et al., 2014). Submesoscale fronts are  
60 challenging to investigate theoretically, observationally, and numerically, owing to the interactions  
61 among multiple dynamical processes and the short spatiotemporal scales of submesoscale fronts  
62 (Barkan et al., 2019). This motivates sustained efforts to identify them robustly using *in situ*  
63 observations, satellite remote sensing, and numerical models.

64 Difficulties in measuring intermittent submesoscale dynamics restrict *in situ* observational  
65 studies to a few well-sampled regions via ship-based measurements (Adams et al., 2017; Poje et  
66 al., 2014; Thompson et al., 2016; Yu et al., 2024) and high-frequency (HF) radar observations  
67 (Archer et al., 2015; Yoo et al., 2018). Recently, Whalen and Drushka (2025) used global ship-  
68 based thermosalinograph measurements to detect submesoscale density fronts, defined as  
69 sequences of at least 4 consecutive points with consistent density gradients exceeding 10<sup>-5</sup> kg m<sup>-4</sup>.  
70 They provide global perspective on the variability and dynamics of submesoscale fronts. However,  
71 the spatiotemporal coverage is limited, with observations predominantly confined to the Northern  
72 Hemisphere and austral summer. They also recognized that some conclusions are not statistically  
73 significant in the Southern Hemisphere. Furthermore, the dataset only provides frontal width and  
74 strength, while lacking information on the frontal area.

75 With the improvement of spatiotemporal coverage and resolution, satellite remote sensing  
76 has been widely used to identify submesoscale fronts. Various diagnostics have been developed to  
77 identify submesoscale fronts, including imagery-based indices, dynamical criteria, and Lagrangian  
78 methods, each emphasizing a different aspect of frontal dynamics. For example, Liu and Levine  
79 (2016) proposed the Heterogeneity Index (HI) to identify surface submesoscale fronts using daily  
80 sea surface temperature (SST) data at 1 km resolution in the North Pacific Subtropical Gyre. HI

81 combines the variance, skewness, and bimodality of SST distributions computed within a  $10 \times 10$   
82 pixel sliding window. Compared to traditional gradient-based methods, HI captures submesoscale  
83 signals while remaining less sensitive to noise. At the same time, an inherent limitation of SST-  
84 based detection is that it may miss fronts with weak thermal contrasts (e.g., salinity-compensated  
85 density fronts). Zhang et al. (2019) defined fronts as locations where the local strain rate is  
86 maximum, with the geostrophic strain rate surpassing  $1.0 \times 10^{-5} \text{ s}^{-1}$  in the direction of the  
87 geostrophic current. Using the dynamical criteria, they directly combined frontal kinematics with  
88 drifter-based chlorophyll tendencies and revealed that ageostrophic secondary circulation induces  
89 upwelling on the light side of fronts, promoting surface chlorophyll growth. Nevertheless, this  
90 approach is constrained by the effective resolution of altimetry-derived geostrophic fields.

91 Lagrangian diagnostics, especially the Finite-Time Lyapunov Exponent (FTLE) and  
92 Finite-Size Lyapunov Exponent (FSLE), have emerged as powerful tools for detecting  
93 submesoscale fronts (Fahlbusch et al., 2024; Guo et al., 2019). Both diagnostics quantify particle  
94 separation in a flow field, but differ in formulation: FTLE quantifies the rate at which initially  
95 neighboring particles separate over a fixed time interval, whereas FSLE is inversely proportional  
96 to the time required for two initially close particles to separate to a prescribed distance (Allshouse  
97 & Peacock, 2015a; d'Ovidio et al., 2004; Hadjighasem et al., 2017; Lapeyre, 2002; Ott, 2002;  
98 Ottino, 1989). Maxima in backward-time FTLE and FSLE (called ridges) correspond to attracting  
99 Lagrangian Coherent Structures (LCSs), which act as transport barriers and often coincide with  
100 surface convergence zones and submesoscale filaments (Gough et al., 2016; Huntley et al., 2011;  
101 Lehahn et al., 2007). These structures delineate frontal boundaries and strongly influence tracer  
102 distribution. Compared to traditional Eulerian diagnostics, Lagrangian methods can uncover  
103 hidden transport structures and fine-scale patterns shaped by chaotic advection, making them  
104 invaluable for studying the complex and transient nature of submesoscale dynamics (Aravind et  
105 al., 2023; Calil & Richards, 2010; Cotté et al., 2015; d'Ovidio et al., 2009; Hernández-Carrasco et  
106 al., 2011). For instance, Guo et al. (2019) used FSLE derived from altimetric velocities to diagnose  
107 submesoscale filaments associated with mesoscale horizontal shear and strain deformation.  
108 However, the ability of FSLE and FTLE to reveal submesoscale transport structures depends on  
109 the accuracy and spatial resolution of the underlying velocity field. Present altimetry or reanalysis  
110 products typically do not fully resolve 1–10 km submesoscale features.

111 Although high-resolution satellite SST and ocean color products can be used to detect  
112 frontal structures, they provide only partial information on frontal dynamics, since temperature  
113 signals may be salinity-compensated (Spiro Jaeger & Mahadevan, 2018), and ocean color is a  
114 nonconservative tracer (Zhang et al., 2019). In contrast, density fronts provide a more reliable  
115 measure of frontal dynamics, directly reflecting water mass boundaries and movement. Density  
116 variations control stratification intensity and determine whether the local water column is  
117 susceptible to vertical perturbations and mixing (Dave & Lozier, 2010). Density fronts are often  
118 associated with frontogenesis, leading to upwelling on the light side and downwelling on the dense  
119 side of the front (McWilliams, 2021). These circulations modify the density profile, regulate the  
120 stratification structure and local stability, and thereby influence the exchange of energy, matter,  
121 and momentum between the surface and subsurface layers (Liu et al., 2025). At present, the limited  
122 availability and coarse resolution of salinity observations hinder direct observation of  
123 submesoscale density structures, motivating the use of high-resolution numerical simulations to  
124 investigate their dynamics and impacts. In such simulations, frontal detection is commonly  
125 achieved by computing horizontal gradients of temperature, salinity, or density and applying fixed

126 thresholds such as the 90th or 95th percentile (e.g., Duan et al., 2024; Spiro Jaeger & Mahadevan,  
127 2018).

128 In this study, we use a realistic, submesoscale-permitting Atlantic Ocean numerical  
129 simulation with kilometer-scale resolution to systematically evaluate and compare commonly  
130 applied diagnostics for identifying submesoscale fronts from observations, including FTLE and  
131 Local Variability Index (LVI, a method based on HI). Specifically, thermal, haline, and density  
132 fronts are identified using LVI computed from surface temperature, salinity, and density fields,  
133 respectively, whereas dynamical fronts are diagnosed using FTLE derived from the surface  
134 velocity field. Previous work has shown that FTLE and FSLE can yield comparable results in  
135 numerical flow visualization when appropriately parameterized (Haller, 2015; Karrasch & Haller,  
136 2013; Peikert et al., 2014). However, FSLE can exhibit jump discontinuities, making it sensitive  
137 and less robust in nonlinear flows and potentially degrading the accuracy of FSLE statistics (Haller,  
138 2023, Section 5.2.8). It can also be ill-posed for certain separation ratios and becomes insensitive  
139 to further flow evolution once the separation time is reached (Karrasch & Haller, 2013). From a  
140 multiscale perspective, FSLE better characterizes large-scale transport structures, whereas FTLE  
141 is more sensitive to finer-scale features of particle dispersion (Boffetta et al., 2001). Moreover,  
142 FTLE is more computationally efficient, as it requires only a single precomputed flow map.  
143 Therefore, we choose FTLE for the subsequent analysis. In addition, we investigate the  
144 relationship between thermal, haline, and dynamical fronts relative to density fronts, to assess the  
145 potential limitations and uncertainties when using satellite SST and velocity fields to identify  
146 fronts.

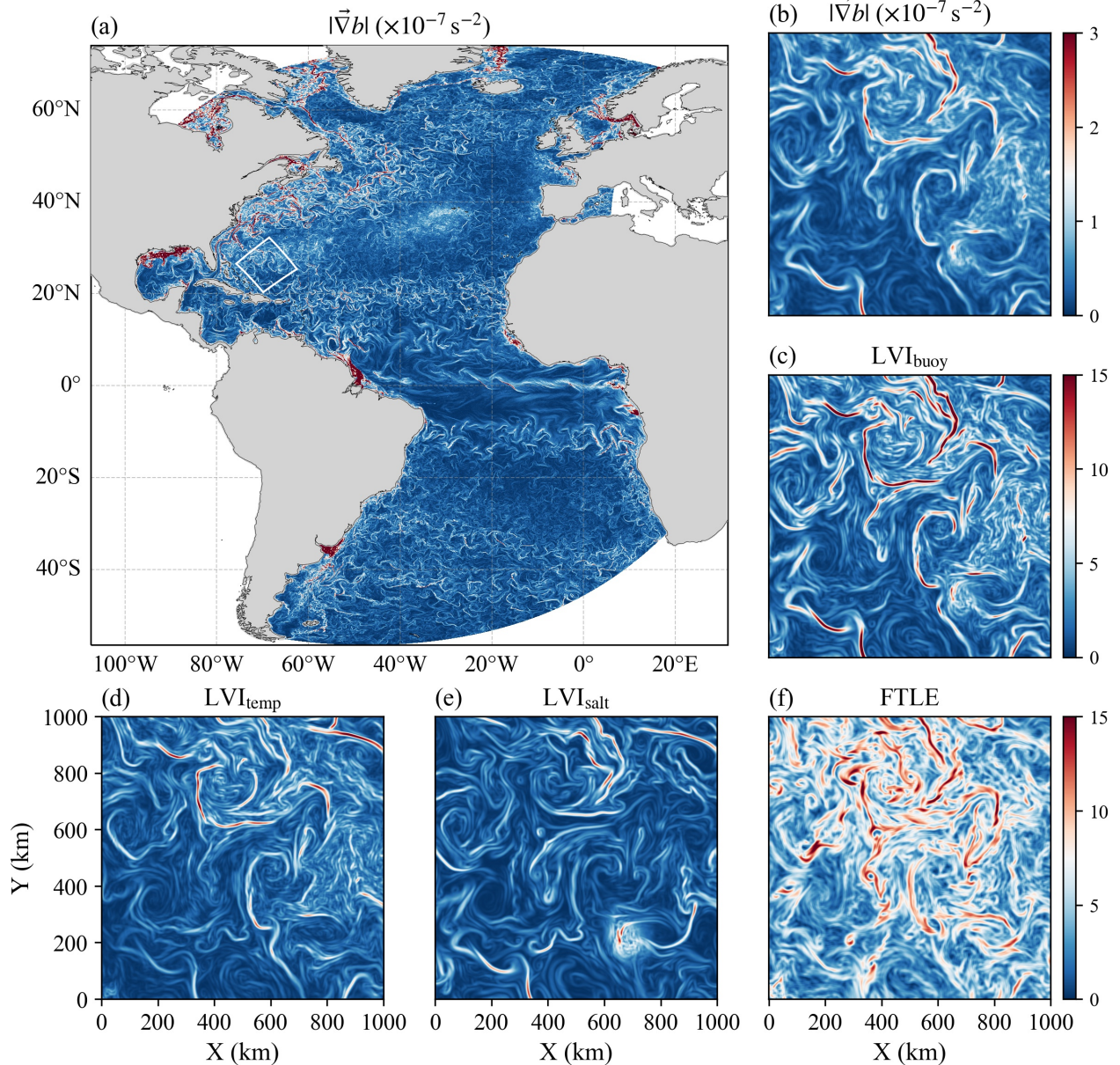
147 This study provides a comprehensive benchmark for surface submesoscale front detection  
148 and reveals regional and seasonal contrasts in frontal occurrence, intensity, and dominant type.  
149 The paper is structured as follows: Section 2 describes the data and methodology; Section 3  
150 presents the characteristics of submesoscale fronts; Section 4 explores the relationship between  
151 dynamical, thermal, and haline fronts relative to density fronts; Section 5 evaluates the  
152 performance of FTLE derived from the coarse velocity; and Section 6 concludes the study.

## 153 **2 Data and methods**

### 154 **2.1 Numerical simulation**

155 This study utilizes the temperature, salinity, and velocity fields of the GIGATL1 simulation  
156 (Gula et al., 2021), a submesoscale-permitting Atlantic-wide (107°W–31°E, 56°S–74°N, Figure 1)  
157 simulation performed with the Coastal and Regional Ocean COmmunity model (CROCO),  
158 developed from the Regional Ocean Modeling System (ROMS; Shchepetkin & McWilliams,  
159 2005). The GIGATL1 grid is orthogonally constructed on an oblique Mercator projection to ensure  
160 near-uniform horizontal spacing. The grid resolution varies from 1 km near the central longitude  
161 to 735 m near the western and eastern edges. The model grid consists of 10,500 points in the zonal  
162 direction and 14,000 points in the meridional direction. It solves the hydrostatic primitive  
163 equations with a free surface, using a terrain-following vertical coordinate system with 100 sigma  
164 levels. The bathymetry is taken from the SRTM30plus dataset (Becker et al., 2009). Vertical  
165 mixing is parameterized using the  $k$ - $\epsilon$  turbulence closure scheme with the Canuto A stability  
166 function (Canuto et al., 2001; Umlauf & Burchard, 2003). No explicit lateral diffusion or viscosity  
167 is used. The effect of bottom friction is parameterized with a logarithmic law of the wall with a  
168 roughness length of 0.01 m.

169           The simulation is run from 02 July 2007 to 05 September 2009. It is initialized using fields  
170 from a coarser 3 km-resolution simulation (GIGATL3), with an identical setup (e.g., Ruan et al.,  
171 2021; Schubert et al., 2025; Vic et al., 2022). Boundary conditions are from Simple Ocean Data  
172 Assimilation (SODA; Carton & Giese, 2008). Atmospheric forcing is provided by hourly fields  
173 from the Climate Forecast System Reanalysis (CFSR), which employ a bulk formulation and a  
174 stress-correction approach to parameterize the feedback of surface ocean currents on the  
175 atmosphere (Renault et al., 2020). Simulations include barotropic tidal forcing at the boundaries,  
176 as well as tidal potential and self-attraction, which are derived from TPXO7.2 and GOT99.2b,  
177 respectively. The GIGATL1 simulation is included in the intercomparison of submesoscale-  
178 permitting realistic ocean models presented in Uchida et al (2022). It has recently been used in  
179 studies investigating mesoscale and submesoscale dynamics, including the ocean kinetic energy  
180 cascade (Schubert et al., 2023), vertical eddy fluxes (de Marez et al., 2025), and the near-inertial  
181 wave dynamics (Qu et al., 2021). For this study, we specifically analyze the daily-averaged  
182 temperature, salinity, and buoyancy of the GIGATL1 simulation for two representative months:  
183 June and December 2008, to examine potential seasonal contrasts in frontal activity. This choice  
184 is justified because a comparison between daily and hourly buoyancy gradients shows only a minor  
185 reduction in intensity, indicating that daily averages still capture the essential frontal signal (Figure  
186 S1). For FTLE discussed in Sections 3, 4, and 5, we used the hourly velocity fields.



187

188 Figure 1. Horizontal buoyancy gradient and frontal diagnostics on 2 June 2008. (a) Horizontal  
 189 buoyancy gradient in the Atlantic Ocean. The buoyancy gradient is defined as  $|\vec{\nabla}b| = g|\vec{\nabla}\rho|/\rho_0$ ,  
 190 where  $b$  is the buoyancy,  $g$  is gravity,  $\rho$  is surface density, and  $\rho_0$  is the reference density of  $1027$   
 191  $\text{kg m}^{-3}$ . The buoyancy gradient is smoothed using a uniform  $11 \times 11$  km filter for comparison with  
 192 the frontal diagnostics. Snapshots in the zoomed-in region of (b)  $|\vec{\nabla}b|$ , (c)  $\text{LVI}_{\text{buoy}}$ , (d)  $\text{LVI}_{\text{temp}}$ , (e)  
 193  $\text{LVI}_{\text{salt}}$ , and (f) FTLE.

194

## 2.2 From Heterogeneity Index to Local Variability Index

195

We apply the HI method proposed by Haëck et al. (2023) to detect surface submesoscale  
 196 fronts. HI is defined as a weighted sum of skewness  $\gamma$ , standard deviation  $\sigma$ , and bimodality  $B$  to  
 197 quantify the heterogeneity of sea surface properties. It is computed at each grid point within an  $11$   
 198  $\times 11$ -pixel square window, corresponding to a spatial scale of approximately 10 km in the

199 GIGATL1 configuration. This scale is chosen to isolate submesoscale features while avoiding  
200 observational noise.

201 Although HI has been applied to satellite SST data to identify thermal fronts, our analysis  
202 reveals two limitations when applied to high-resolution simulations. First, the bimodality  
203 component strongly depends on the choice of histogram bin width. Sensitivity analyses show that  
204 different bin widths yield substantial variations in the number, intensity, and continuity of detected  
205 fronts, rendering the bimodality component unreliable for quantitative front identification (see  
206 Text S1). Second, skewness tends to highlight isolated anomalies and small-scale noise, producing  
207 patchy or duplicated structures rather than coherent fronts (Text S2).

208 To avoid these issues, we apply a simplified and more robust metric, the Local Variability  
209 Index

$$\begin{aligned} \text{LVI}_\phi &= a \times (b\sigma_\phi), \text{ with} \\ a &= \frac{9.5}{P_{95}(b\sigma_\phi)}. \end{aligned} \quad (1)$$

211  $\sigma_\phi$  denotes the standard deviation of tracer  $\phi$  within a sliding window centered at each pixel.  $b$  is  
212 the inverse of the standard deviation of  $\sigma_\phi$  over the entire domain and full time series.  $P_{95}$  is the  
213 95th percentile.  $a$  is a normalization coefficient to ensure that 95% of  $\text{LVI}_\phi$  values are less than  
214 9.5. LVI retains only the standard deviation component, which reliably captures coherent frontal  
215 structures across all tracers and is less affected by binning artifacts or outliers (Figure S5).  
216 Additionally, it reflects the magnitude of local variability, which is closely related to submesoscale  
217 gradients. Figure S7 compares LVI of buoyancy ( $\text{LVI}_{\text{buoy}}$ ) with the buoyancy gradient. After  
218 smoothing the gradient at the same 10-km scale, the two fields exhibit nearly identical spatial  
219 patterns, with a Pearson correlation coefficient  $r = 0.97$  computed over all data points.

220 Although LVI shows high spatial correspondence with smoothed gradients, it is adopted in  
221 this study because it directly targets variability at a prescribed spatial scale ( $\sim 10$  km), is more  
222 robust to observational noise, and provides a normalized framework for consistent comparison  
223 across different tracers. To further examine the performance of LVI and gradient-based methods  
224 to noise, we add Gaussian noise to the buoyancy field (Figure S8a).  $|\vec{\nabla}b|$  becomes strongly  
225 contaminated by noise, as finite-difference gradients amplify the grid-scale variability (Figure S8c).  
226 Even after spatial filtering is applied to  $|\vec{\nabla}b|$ , residual small-scale artifacts remain, and the fronts  
227 detected from the filtered noisy  $|\vec{\nabla}b|$  are more fragmented and show noisier edges than those  
228 obtained from the original field (Figure S9a). However,  $\text{LVI}_{\text{buoy}}$  is less sensitive to noise and better  
229 preserves coherent frontal patterns (Figure S9b). When the diagnostics are computed from the  
230 filtered buoyancy field, the spatial correspondence between the gradient-based and LVI-based  
231 diagnostics becomes much closer (Figures S9c and S9d). However,  $|\vec{\nabla}b|$  computed from the  
232 filtered noisy buoyancy field still retains residual small-scale variability, and the detected frontal  
233 boundaries are less smooth and less continuous than those obtained from LVI. This comparison  
234 further supports the use of LVI as a scale-aware diagnostic that better preserves coherent frontal  
235 structures in noisy buoyancy fields.

236 As a scale-dependent metric, LVI may weaken filament signatures that are much sharper  
237 than the chosen window. Accordingly, the window size should be adjusted to the target scale. In  
238 this study, submesoscale fronts are identified at a spatial scale of  $\sim 10$  km scale, representative of

239 typical satellite resolution. Thus, the applicability of LVI depends on high-resolution data that  
 240 resolve submesoscale variability. The LVI method is applied to sea surface temperature, salinity,  
 241 and density fields to detect thermal, haline, and density fronts, respectively (Figures 1c–1e and  
 242 S10). This consistent framework enables comparative analyses of frontal structures across multiple  
 243 tracers at submesoscale resolution.

### 244 2.3 Finite-Time Lyapunov Exponent

245 Backward-time and forward-time FTLE ridges indicate the directions of horizontal  
 246 stretching and compressing, respectively (Figure S11). The backward-time FTLE often delineates  
 247 attracting LCSs, which are associated with surface frontal zones characterized by convergent flows  
 248 and tracer sharpening. Therefore, in this study, the term “FTLE” will correspond to backward-time  
 249 FTLE, unless specified otherwise.

250 At each initial position  $\mathbf{x}_0$  and time  $t_0$ , the FTLE over an integration time  $T$  is defined as

$$251 \text{FTLE}_{t_0}^{t_0-T}(\mathbf{x}_0) = \frac{1}{|T|} \ln \left( \sqrt{\lambda_{\max}(\mathbf{C}(\mathbf{x}_0, t_0))} \right), \quad (2)$$

252 where  $\lambda_{\max}$  is the maximum eigenvalue of the right Cauchy–Green deformation tensor,

$$253 \mathbf{C}(\mathbf{x}_0, t_0) = \left( \nabla \Phi_{t_0}^{t_0-T}(\mathbf{x}_0) \right)' \nabla \Phi_{t_0}^{t_0-T}(\mathbf{x}_0). \quad (3)$$

254  $\Phi_{t_0}^{t_0-T}$  is the flow map which gives the final position of a fluid parcel initially at  $\mathbf{x}_0$ , after advection  
 255 over the time interval  $T$ . The flow map depicts the flow directions and particle movements.

256 To construct the flow map  $\Phi_{t_0}^{t_0-T}$ , we use a standard fourth-order Runge–Kutta method with  
 257 an integration time step  $dt = 2$  h. We obtained the trajectory  $\mathbf{x}(t_0 - T)$  by numerically integrating  
 258 the velocity field over a finite time interval  $T = 24$  h. We interpolate the velocity data using a  
 259 bilinear interpolation scheme. The interpolation procedure for the non-uniform grid is described  
 260 in Text S3.

261 To compute the gradient of the flow map  $\nabla \Phi$ , four auxiliary particles are initialized at an  
 262 initial separation  $\delta_0 = 1$  km in the  $x$  and  $y$  directions around each particle, which is located at  $\mathbf{x}_0$  at  
 263 time  $t_0$ . Their final positions after advection are used to compute the components of the Jacobian  
 264 matrix via centered finite differences,

$$265 \nabla \Phi = \begin{bmatrix} \frac{x^{(+\delta_x)} - x^{(-\delta_x)}}{2\delta_x} & \frac{x^{(+\delta_y)} - x^{(-\delta_y)}}{2\delta_y} \\ \frac{y^{(+\delta_x)} - y^{(-\delta_x)}}{2\delta_x} & \frac{y^{(+\delta_y)} - y^{(-\delta_y)}}{2\delta_y} \end{bmatrix}, \quad (4)$$

266 where superscripts indicate perturbations in the X and Y directions, respectively. Instead of using  
 267 the four nearest neighbors on the computational grid, this setup ensures accurate estimation of the  
 268 local deformation gradient, especially under non-uniform grid configurations. For example, as  
 269 shown in Figure S12, computing  $\nabla \Phi$  using auxiliary particles with fixed spacing yields clearer  
 270 filamentary and vortical structures. The boundaries of submesoscale features appear sharper and  
 271 fine structural details are more evident. However, using the nearest grid-based neighbors produces  
 272 more blurred structural boundaries, with many small-scale eddies and fronts being smoothed out  
 273 and less discernible.

274 FTLE is sensitive to the initial separation distance  $\delta_0$ . In theory, FTLE describes the  
 275 separation of infinitesimally close particles over a finite time. In practice, however,  $\delta_0$  cannot be  
 276 chosen arbitrarily small in discrete velocity fields (Günther et al., 2016). Numerical errors are  
 277 observed when the auxiliary grid points used for the finite-difference approximation become  
 278 indistinguishable from each other (Allshouse & Peacock, 2015b). The effects of numerical errors  
 279 and velocity noise on FTLE fields become more pronounced at smaller grid spacing (Fang et al.,  
 280 2020). Such effects are also observed in this study. For example, when  $\delta_0 = 0.0001$  km, the  
 281 resulting FTLE fields exhibit typical “salt-and-pepper” noise (Rafiee & Farhang, 2023),  
 282 characterized by isolated pixel-scale dots superimposed on the FTLE field (Figure S13a).  
 283 Conversely, choosing a too large  $\delta_0$  smears FTLE ridges and reduces spatial resolution (Figure  
 284 S13c). Balancing these considerations, we set  $\delta_0 = 1$  km for the FTLE calculations, which is  
 285 consistent with the approximate spatial resolution of the velocity field. This choice suppresses  
 286 numerical errors while preserving the main LCSs, yielding results comparable to those obtained  
 287 using  $\delta_0 = 0.0001$  km when the salt-and-pepper noise is ignored.

288 Another key scale factor affecting the FTLE is the integration time  $T$ , because FTLE  
 289 quantifies the particle separation rate over a finite time interval. When  $T$  is short, the FTLE captures  
 290 local, short-term stretching, making it sensitive to small-scale, transient flow features. As  $T$   
 291 increases, fine-scale structures become weaker and less distinct. For example, at  $T = 120$  h, only  
 292 strong, persistent structures remain, while finer details are smoothed out (Figure S13f). Therefore,  
 293 we choose an intermediate value,  $T = 24$  h, to retain submesoscale filaments while suppressing  
 294 short-lived noise.

295 Apart from the integration duration, the integration time step  $dt$  used during trajectory  
 296 computation also affects the FTLE results. A large  $dt$  (e.g., 10h) may introduce integration errors,  
 297 leading to spatial distortions, as shown in Figure S13i. Even though the velocity field is available  
 298 at 1 h intervals, we choose an integration time step  $dt = 2$  h to reduce computation time while  
 299 achieving accurate resolution of submesoscale structures (Figure S13h). Additionally, the mean  
 300 difference in final positions between  $dt = 2$  h and  $dt = 1$  h is 0.94 km. When normalized by the  
 301 trajectory length, the mean trajectory error is 0.045. These errors are small relative to the grid  
 302 spacing (1 km) and the target submesoscale scale ( $\sim 10$  km), indicating that  $dt = 2$  h provides  
 303 sufficient numerical accuracy. Details of this assessment are described in Text S4.

304 To extract frontal structures at 10-km scales, we first smooth the FTLE fields using an  $11$   
 305  $\times 11$  mean filter, consistent with the LVI computation. Then we normalize the FTLE values  
 306 following the same procedure (used for LVI),

$$307 \quad \text{FTLE}_{norm} = a \times (b \times \mu_{\text{FTLE}}), \quad b = \frac{1}{\text{std}(\mu_{\text{FTLE}})_{\text{total}}}, \quad a = \frac{9.5}{P_{95}(b \times \mu_{\text{FTLE}})}, \quad (5)$$

308 where  $\mu_{\text{FTLE}}$  denotes the mean FTLE value within an  $11 \times 11$  window.  $(\mu_{\text{FTLE}})_{\text{total}}$  includes all  
 309 FTLE data across the study period, and  $P_{95}$  is the 95th percentile of  $b \times (\mu_{\text{FTLE}})_{\text{total}}$ , indicating that  
 310 95% of FTLE values fall below 9.5. FTLE discussed in Sections 3, 4, and 5 is the normalized  
 311 FTLE. A zoomed snapshot of the FTLE field is shown in Figure 1f.

## 312 2.4 Submesoscale front extraction using filters and thresholds

313 Previous studies often relied on fixed thresholds to delineate frontal regions (Barceló-Llull  
 314 et al., 2021; Duan et al., 2024). However, when applied over basin scales, such thresholds may fail

315 in areas where tracer gradients are particularly strong. To resolve this issue, we progressively apply  
 316 temporal and spatial filters to isolate submesoscale signals. Figure S15 compares the effects of  
 317 thresholding, temporal filtering, spatial filtering, and their combination on frontal extraction.

318 Using the Mississippi Plume as an example, applying the 90th-percentile  $LVI_{buoy}$  threshold  
 319 derived from the whole Atlantic produces unrealistically broad frontal zones that obscure the  
 320 narrow, filamentary morphology characteristic of submesoscale fronts (Figure S15a). We perform  
 321 a temporal filtering on  $LVI_{buoy}$  by removing a 10-day moving average (Wang et al., 2025). Positive  
 322 anomalies from the temporally filtered field are used as a mask for the original  $LVI_{buoy}$ , suppressing  
 323 weak signals and improving frontal-zone localization (Figure S15b). However, the boundaries  
 324 between adjacent fronts remain indistinct. A high-pass Gaussian spatial filter (cutoff = 30 km)  
 325 improves this separation by removing mesoscale contributions (Figure S15c). Previous studies  
 326 commonly apply spatial filtering after temporal filtering (Gaube et al., 2015; Liu et al., 2023). One  
 327 can see that the result of this combined filtering is similar to spatial filtering but retains more  
 328 residual weak signals (Figure S15d).

329 Temporal and spatial filtering act to extract signals at specific scales in their respective  
 330 dimensions. To ensure consistency with submesoscale temporal and spatial scales, we construct  
 331 two masks from the time- and space-filtered fields ( $>0$ ). In addition, we introduce a third mask  
 332 based on the 90th-percentile threshold to identify regions of strong tracer gradients (Figure S15f).  
 333 We also apply the same masking strategy to LVI of temperature ( $LVI_{temp}$ ), LVI of salinity ( $LVI_{salt}$ ),  
 334 and FTLE (Figure S16).

### 335 **3 Characteristics of submesoscale fronts in the Atlantic Ocean**

#### 336 **3.1 Submesoscale diagnostics and frontogenesis**

337 To investigate how dynamical processes deform the flow and generate fronts, we analyze  
 338 key kinematic quantities derived from the velocity gradient tensor, including the strain magnitude  
 339 ( $\sigma$ ), vertical vorticity ( $\zeta$ ), and horizontal divergence ( $\delta$ ), defined as:

$$340 \quad \sigma = \sqrt{(u_x - v_y)^2 + (v_x + u_y)^2}, \quad \zeta = v_x - u_y, \quad \delta = u_x + v_y. \quad (6)$$

341 In this study,  $\sigma$  and  $\delta$  are normalized by the absolute value of Coriolis frequency  $|f|$ , and  $\zeta$  is  
 342 normalized by  $f$ . Normalization provides dimensionless measures relative to planetary rotation,  
 343 highlighting the role of ageostrophic motions in governing frontal evolution (Capet et al., 2008;  
 344 Luko et al., 2023; Napolitano et al., 2021; Shcherbina et al., 2013; Thomas et al., 2008).

345 Frontogenesis describes the intensification of horizontal density gradients, which can be  
 346 driven by mesoscale straining, the nonlinear evolution of baroclinic instability, and interactions  
 347 between fronts and small-scale turbulence within the turbulent thermal wind (TTW) framework  
 348 (Gula et al., 2014; McWilliams et al., 2015; Sullivan & McWilliams, 2018; Verma et al., 2019).  
 349 The frontogenesis function quantifies this process by isolating the advective contribution to the  
 350 evolution of the horizontal buoyancy gradient (Hoskins, 1982),

$$351 \quad F = \frac{D|\nabla_h \rho|}{Dt}, \quad (7)$$

352 which can be separated into several components. In this study, we neglect the vertical advection  
 353 and diabatic terms, focusing on the horizontal advective contribution.

354 
$$F_h = \mathbf{Q}_h \cdot \nabla_h \rho, \text{ with} \tag{8}$$

$$\mathbf{Q}_h = -\left(u_x \rho_x + v_x \rho_y\right) \mathbf{i} - \left(u_y \rho_x + v_y \rho_y\right) \mathbf{j}.$$

355 In (8),  $F_h$  quantifies the growth rate of the horizontal density gradient driven by horizontal straining.  
 356 To better compare frontogenesis efficiency across different regions and tracers, we define a  
 357 nondimensional measure following Barkan et al. (2019),

358 
$$F_s = \frac{F_h}{f |\nabla_h \rho|^2}. \tag{9}$$

359 Positive  $F_s$  correspond to frontogenesis, while negative values correspond to frontolysis.  
 360 To avoid the singularity as the Coriolis parameter approaches zero, regions within  $\pm 5^\circ$  latitude are  
 361 excluded from the analysis.

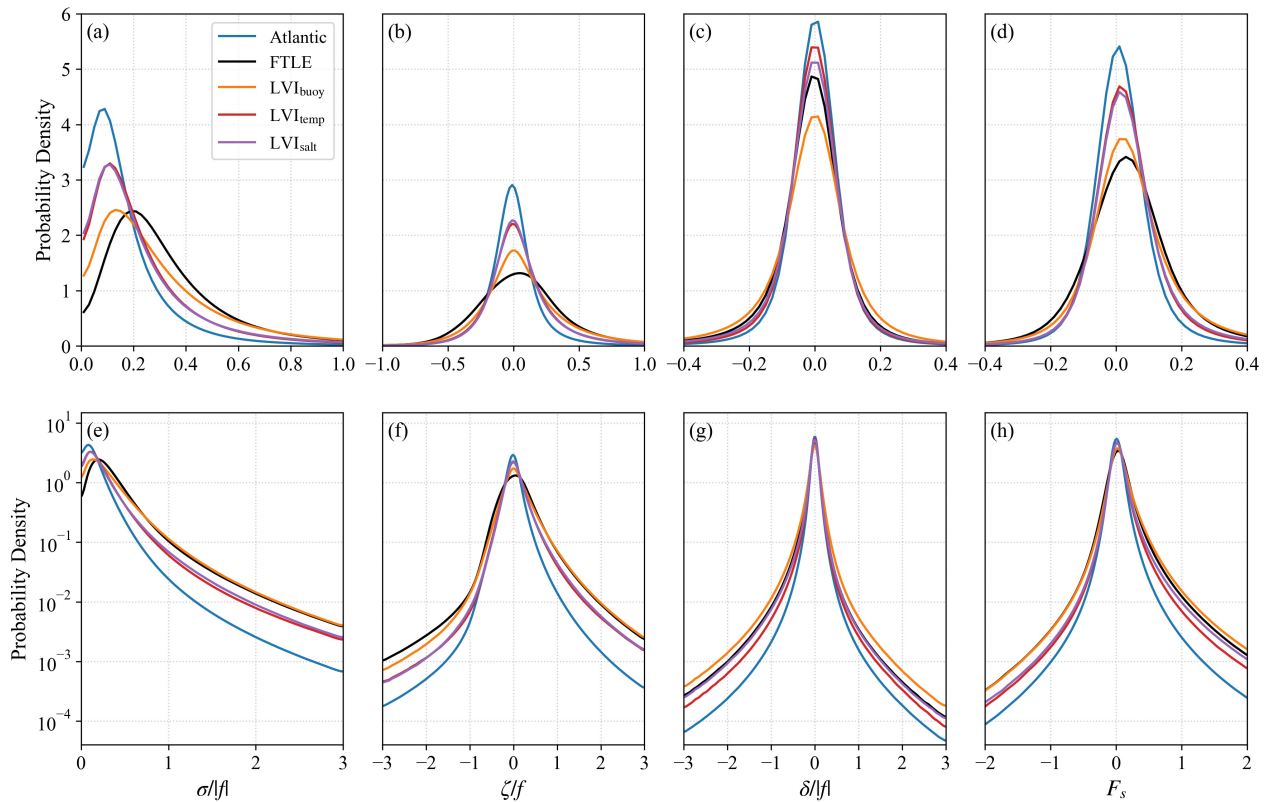
362 We calculate the probability density functions (pdfs) of  $\sigma/|f|$ ,  $\zeta/f$ ,  $\delta/|f|$ , and  $F_s$  over different  
 363 frontal zones, including dynamical (FTLE), density (LVI<sub>buoy</sub>), thermal (LVI<sub>temp</sub>), and haline  
 364 (LVI<sub>salt</sub>) fronts. Compared to the background, all front types have stronger dynamical activity,  
 365 including deformation, rotation, convergence/divergence, and frontogenesis/frontolysis (Figure 2).  
 366 Moreover, there is a distinct asymmetry skewed toward cyclonic vorticity, convergence, and  
 367 frontogenesis, which is a typical feature of submesoscale fronts (Balwada et al., 2021; Barkan et  
 368 al., 2019; Hetland et al., 2025; Shcherbina et al., 2013). The cyclonic skewness of the vorticity  
 369 field arises from the asymmetric response of vorticity tendency to vortex stretching (McWilliams,  
 370 2016). These results are consistent with previous studies, suggesting that FTLE and LVI are  
 371 capable of capturing key dynamical features of submesoscale fronts.

372 Compared to thermal and haline fronts, dynamical and density fronts occur more frequently  
 373 at larger strain, vorticity, convergence, and frontogenesis ( $\sigma$ ,  $\zeta$ ,  $-\delta$ , and  $F_s \geq f$ ; Figure 2). This  
 374 reflects their closer link to deformation-driven frontogenesis. In strain-dominated and confluent  
 375 regions, cross-front compression and along-front stretching sharpen lateral buoyancy gradients,  
 376 promoting the formation of density fronts (Hoskins, 1982). Additionally, convergence in such  
 377 regions facilitates the aggregation of tracer gradients and enhances the vertical tilt of isopycnals,  
 378 both of which contribute to frontogenesis. The cross-front compression, convergence, and  
 379 horizontal shear can enhance vertical relative vorticity (Figure 2f). With FTLE consistently  
 380 capturing these features as attracting LCSs, the pdfs of density and dynamical fronts show a nearly  
 381 identical distribution when  $\sigma/|f| > 0.5$  and  $\zeta/f > 0.2$ . By contrast, thermal and haline fronts are more  
 382 directly influenced by surface forcing processes such as heating, evaporation, precipitation, and  
 383 vertical mixing. Moreover, frontal dynamics are primarily governed by density, whereas the  
 384 dynamical relevance of thermal and haline fronts depends on their contributions to density  
 385 variations. When density compensation in opposing temperature and salinity gradients reduces the  
 386 net buoyancy gradients (Rudnick & Ferrari, 1999), frontogenesis and the associated dynamical  
 387 response are weakened.

388 Interestingly, the probability density distribution of dynamical fronts is more similar to  
 389 haline fronts than density fronts in terms of  $\delta/|f|$  (Figure 2g). This similarity may reflect the fact  
 390 that haline fronts are typically more surface-intensified, primarily influenced by evaporation–  
 391 precipitation, river discharge, or sea ice melting and freezing, and are often associated with shallow  
 392 mixed layers where salinity dominates density (Spiro Jaeger & Mahadevan, 2018; Wilson & Riser,

393 2016). These fronts tend to produce strong and persistent surface convergence, which may be well  
 394 captured by the time-integrated nature of FTLE.

395



396

397 Figure 2. Probability density functions of normalized dynamical variables for the Atlantic  
 398 background (blue) and submesoscale fronts identified by FTLE (black),  $LVI_{buoy}$  (orange),  $LVI_{temp}$   
 399 (red), and  $LVI_{salt}$  (purple). (a–d) use a linear y-axis, and (e–h) use a logarithmic y-axis. Variables:  
 400 (a, e)  $\sigma/|f|$ , (b, f)  $\zeta/f$ , (c, g)  $\delta/|f|$ , (d, h)  $F_s$ .

401

### 3.2 Spatial and seasonal patterns of submesoscale fronts in the Atlantic Ocean

402

403 We distinguish four complementary diagnostics that represent different aspects of  
 404 submesoscale frontogenesis. Density fronts ( $LVI_{buoy}$ ) provide the most direct measure of  
 405 submesoscale frontal dynamics. Thermal and haline fronts ( $LVI_{temp}$ ,  $LVI_{salt}$ ) identify regions where  
 406 buoyancy gradients are primarily dominated by temperature or salinity. Dynamical fronts  
 407 diagnosed by backward-time FTLE correspond to Lagrangian stretching and convergence  
 408 structures that can sharpen existing thermohaline gradients. Thermohaline compensation, together  
 409 with the potential spatial offset between FTLE structures and buoyancy-gradient sharpening, can  
 410 result in substantial differences in the spatial coverage and seasonal variability among different  
 411 front types. As a baseline for the subsequent co-occurrence and performance analyses, we quantify  
 412 the frequency and strength of each front type across the Atlantic. Seasonal means are computed  
 413 with summer defined as June in the Northern Hemisphere and December in the Southern  
 414 Hemisphere, and vice-versa for winter. Here, frequency is defined as the fraction of days with  
 415 detected fronts relative to the total number of days in a given month. The strength of fronts  
 416 represents the monthly mean of the magnitude of  $LVI_{\phi}$  and FTLE, normalized beforehand and  
 therefore a dimensionless quantity.

417 Regions where all front types co-occur with both high frequency and intensity are primarily  
418 observed in western boundary currents and extensions, such as the Gulf Stream (GS) and the  
419 Brazil–Malvinas Confluence (BMC), as well as in the subtropical–subpolar transition zones  
420 (Figures 3). These regions coincide with stronger tracer gradients, velocity speed, strain, and  
421 relative vorticity (absolute value) than the tropical Atlantic interior (about 10°–20°N/S) and  
422 subtropical gyre interiors (20°–40°N/S; Figures S18 and S19; Table S1). In the latter regions,  
423 fronts of all types are sparse.

424 Dynamical fronts are widespread across the basin, with high frequencies not only in regions  
425 where all front types co-occur but also in the Benguela Current, Canary Current, the equatorial  
426 currents, and the subtropical gyre (Figures 3a and 3b). Compared to thermal and haline fronts,  
427 dynamical fronts show broader spatial coverage and more uniform intensity patterns (Figures S17a  
428 and S17e). This is because FTLE identifies attracting transport structures associated with flow  
429 deformation and particle convergence. Such deformation can be generated by mesoscale currents  
430 and eddies, while their convergent branches promote tracer accumulation and frontal sharpening  
431 at smaller scales (Chelton et al., 2011; McWilliams, 2016). Consequently, dynamical fronts are  
432 less constrained by local temperature and salinity contrasts than thermal and haline fronts.

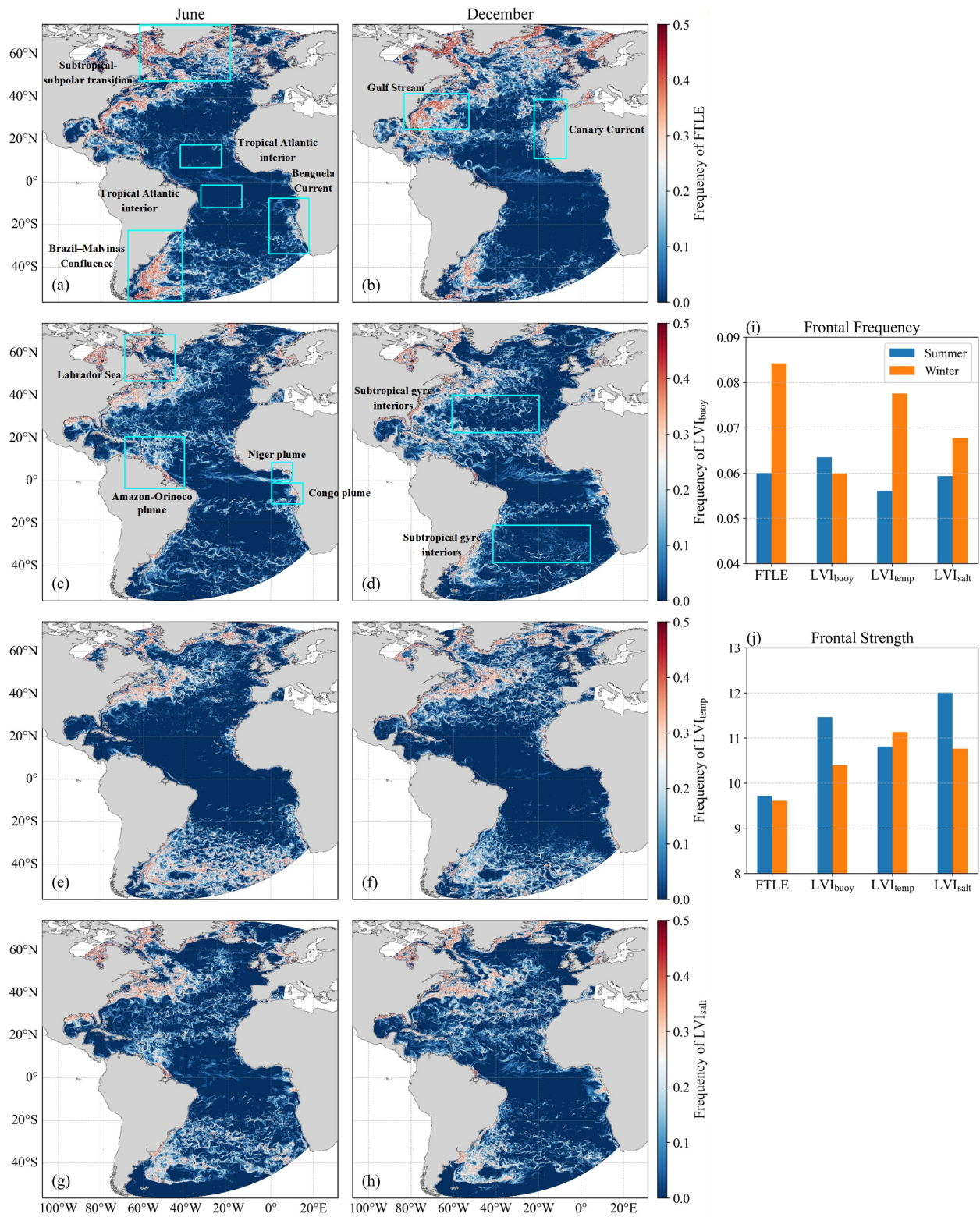
433 Density fronts are also concentrated near coastal regions and river-influenced zones (e.g.,  
434 the Amazon–Orinoco, Congo, and Niger plumes), where buoyancy gradients driven by freshwater  
435 input dominate (Figures 3c and 3d). Their intensities are strongest in the Labrador Sea, the North  
436 Atlantic subpolar gyre, and along western boundary current extensions (Figures S17b and S17f).  
437 In the Labrador Sea and subpolar gyre, strong density fronts are supported by large-scale  
438 thermohaline structure associated with deep convection and water-mass transformation. Along  
439 western boundary current extensions, enhanced frontal intensity is more directly linked to  
440 mesoscale strain and frontogenesis driven by eddy–jet interactions.

441 The most distinctive feature of thermal fronts is their sparse distribution between 20°S and  
442 20°N, which is consistent with the observational thermal fronts (Figure 4c in Xing et al., 2025). In  
443 the equatorial zone, uniform heating from intense solar shortwave radiation homogenizes the  
444 surface temperature field (Xing et al., 2023). Strong stratification and weak mesoscale dynamics  
445 suppress the development of thermal fronts (Liu et al., 2021). In contrast, strong thermal fronts are  
446 observed in the North Atlantic Subpolar Front Zone (45°–60°N), the South Atlantic westerly wind  
447 belt (40°–55°S), and along the eastern boundary upwelling regions such as the southwestern  
448 African coast and the Canary Current. These pronounced fronts result from sharp thermal contrasts  
449 between distinct water masses, which arise from different physical processes across regions. In  
450 subpolar regions, strong horizontal temperature gradients are maintained by large-scale circulation  
451 and intense air–sea heat fluxes (Large & Yeager, 2009; Marshall & Schott, 1999). In the Southern  
452 Ocean, sharp thermal gradients are shaped by strong wind forcing and further intensified by strain  
453 associated with energetic mesoscale eddy activity (Liu et al., 2021; Thomas et al., 2008). Along  
454 eastern boundary systems, coastal upwelling brings cold subsurface waters to the surface,  
455 generating strong cross-shore temperature contrasts (Bakun, 1990; Chavez & Messié, 2009).

456 Unsurprisingly, haline fronts show strong spatial correspondence with major river outflows  
457 and regions of evaporative concentration (Figures 3g and 3h). A prime example is the Amazon–  
458 Orinoco River plume, where a pronounced haline front forms at the river–ocean interface (Coadou-  
459 Chaventon et al., 2024). Here, the front is particularly strong and frequent during the boreal  
460 summer (June), reflecting the seasonal peak in river discharge. The freshwater is advected  
461 northwestward along the Brazilian Shelf by the North Brazilian Current and spread outward into

462 the western equatorial Atlantic Ocean. In December, the Amazon plume weakens, and haline  
463 fronts become more dominant in subtropical gyres where evaporation drives salinity gradients.

464         Seasonal variability of frontal frequency and strength further reflects how different frontal  
465 diagnostics respond to seasonal forcing (Figures 3i and 3j). Dynamical fronts exhibit a pronounced  
466 seasonal cycle in frequency, with substantially higher occurrence during winter than summer,  
467 reflecting the enhancement of mesoscale and submesoscale stirring due in part to the presence of  
468 mixed-layer instabilities under stronger wintertime forcing (Callies et al., 2015). Their intensity,  
469 however, remains relatively stable across seasons. For thermal fronts, the frequency increases  
470 significantly during winter, due to intensified surface cooling that enhances near-surface  
471 temperature gradients. Regions with shallower mixed layers cool more efficiently than deeper-  
472 mixed-layer regions, increasing the cross-front temperature contrast (Spiro Jaeger & Mahadevan,  
473 2018; Tozuka & Cronin, 2014). Haline fronts show higher frequency and strength during summer  
474 near riverine sources but become more widespread during winter in subtropical regions as  
475 evaporation strengthens. Density fronts are slightly more active and stronger in summer, but  
476 mostly in coastal regions due to the seasonal cycle of freshwater input.



479 Figure 3. Spatial and seasonal variations of submesoscale fronts in the Atlantic Ocean. (a–h)  
 480 Spatial distributions of frontal occurrence frequency in the Atlantic Ocean during June and  
 481 December. Rectangular boxes and text labels indicate the major currents and regions discussed in

482 the text. Seasonal mean (i) frontal frequency and (j) frontal strength. Blue bars represent summer  
483 and orange bars represent winter. Frequency is defined as the ratio of the number of days with  
484 detected fronts to the total number of days in each month. Strength is the monthly mean of  
485 normalized frontal intensity, a dimensionless quantity.

#### 486 **4 Relationships of dynamical, thermal, and haline fronts to density fronts**

487 Submesoscale fronts develop through frontogenesis, which can be driven by multiple  
488 processes, including deformation by mesoscale strain fields and mixed-layer instabilities. In  
489 general, density and dynamical fronts represent complementary aspects of submesoscale  
490 frontogenesis. Density fronts diagnosed by LVI indicate the sharpening of lateral buoyancy  
491 gradients, whereas dynamical fronts diagnosed by backward-time FTLE identify attracting LCSs  
492 and deformation zones associated with strong strain and convergence.  $LVI_{temp}$  and  $LVI_{salt}$  represent  
493 subsets of density fronts, depending on whether temperature or salinity dominates the local density  
494 variability. However, these indicators may detect fronts that do not correspond to true density  
495 fronts, when temperature and salinity gradients compensate each other. FTLE may capture strong  
496 strain fields associated with eddies that have weak relative vorticity and do not necessarily  
497 correspond to submesoscale fronts (Shcherbina et al., 2013).

498 In this Section, we quantify the regions and conditions under which density fronts  
499 diagnosed by  $LVI_{buoy}$  are captured by FTLE,  $LVI_{temp}$ , and  $LVI_{salt}$ , individually or in combination,  
500 as well as the conditions under which these diagnostics produce false detections. We also examine  
501 the mechanisms responsible for the different frontal configurations.

##### 502 **4.1 Diagnostic framework**

503 To quantify the relationship among dynamical, thermal, haline and density fronts, we  
504 classify the dominant co-occurrence patterns of these fronts relative to density fronts. For each  
505 grid point, only occurrences exceeding 1 day were considered. A front is classified as dominant  
506 when its frequency exceeds that of the second most frequent front by more than 5 days. In  
507 coexistent regions, all fronts with frequencies within 5 days of the maximum are considered  
508 simultaneously active, and their combinations were used to define the co-occurrence types. Since  
509 dynamical, density, thermal, or haline fronts rarely occur alone, we exclude these isolated  
510 categories and consider only types coexistent with density fronts to build the categorical maps  
511 (Figures 4, 6, and S20). Percentages were calculated as the fraction of each category area relative  
512 to all classified categories area.

513 In addition, to quantify the relative importance of thermal expansion and haline contraction  
514 effects on density, we calculate the horizontal Turner angle:

$$515 \quad Tu_x = \arctan\left(\frac{\alpha T_x}{\beta S_x}\right), \quad (10)$$

516 where  $\alpha$  is the thermal expansion coefficient,  $\beta$  is the haline contraction coefficient,  $T_x$  and  $S_x$  are  
517 the gradients of temperature and salinity in the zonal direction. Based on  $Tu_x$ , gradients can be  
518 classified in four types:  $Tu_x$  in  $[-\pi/2, -\pi/4]$  indicates Temperature-controlled Reinforcement  
519 (TR);  $Tu_x$  in  $[-\pi/4, 0]$  indicates Salinity-controlled Reinforcement (SR);  $Tu_x$  in  $[0, \pi/4]$  indicates  
520 Salinity-controlled Compensation (SC);  $Tu_x$  in  $[\pi/4, \pi/2]$  indicates Temperature-controlled  
521 Compensation (TC). The meridional Turner angle ( $Tu_y$ ), shows a similar spatial distribution  
522 (Figure S21).

523 To characterize how closely each indicator overlaps with density fronts locally, we  
 524 compute the F1-score within a 201×201 pixel sliding window. The F1-score is mathematically  
 525 equivalent to the Dice coefficient, a metric often used in segmentation tasks for measuring spatial  
 526 overlap between two binary maps. In practice, F1-score is defined as the harmonic mean of  
 527 precision and recall,

$$528 \quad \text{F1-score} = \frac{2|X \cap Y|}{|X| + |Y|}, \quad (11)$$

529 which avoids a bias toward under-detection or over-detection. In Equation 11, X denotes the sets  
 530 of detected fronts ( $LVI_{\text{temp}}$ ,  $LVI_{\text{salt}}$ , and FTLE) and Y represents the reference density fronts  
 531 ( $LVI_{\text{buoy}}$ ).

#### 532 4.2 Widespread density fronts associated with dynamical and thermohaline fronts

533 The distribution of dominant frontal types shows strong regional contrasts across the  
 534 Atlantic Ocean (Figure S20). The most widespread co-occurrence patterns are those in which  
 535 density fronts occur together with FTLE and at least one tracer-based front, namely “All”, “FTLE  
 536 + Temp”, and “FTLE + Salt”, which together account for 51.5% of all classified category area in  
 537 Figure S20. These combinations are expected due to the nature of frontogenetic processes.

538 All four types of fronts are simultaneously active along major boundary currents, including  
 539 the GS and its extension, the North Atlantic Drift, the Brazil Current, the Canary Current, the  
 540 Greenland Current, and the Labrador Current (Figures 4a and 4b). These regions also exhibit high  
 541 F1-scores for all three indicators relative to density fronts (Figure 5). These zones are characterized  
 542 by intense strain and sharp horizontal thermohaline gradients (Figures S18 and S19), creating  
 543 favorable conditions for the emergence and interaction of all types of fronts. In the “All” region,  
 544 SC and TC types each account for ~40%, indicating that density fronts in this region are primarily  
 545 compensated. For example, the Gulf Stream advects warm, saline water northward and encounters  
 546 cold, fresh subpolar water, thereby compensating for the density gradient (Figure S19).  
 547 Compensation does not necessarily imply a weak buoyancy gradient. The buoyancy gradient  
 548 magnitude

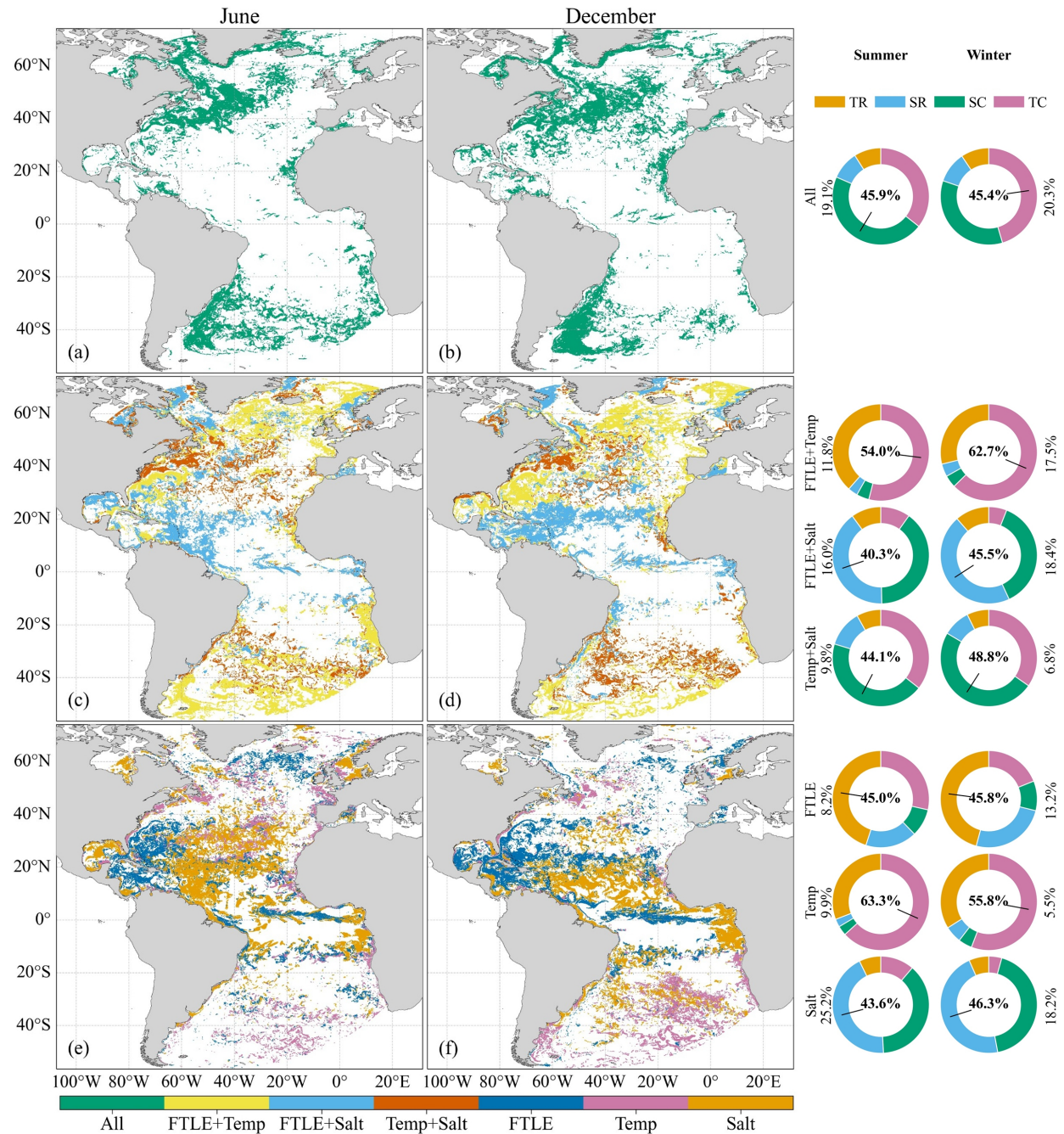
$$549 \quad |\nabla b| = g|\alpha \nabla T - \beta \nabla S|, \quad (12)$$

550 can still be substantial when the residual between thermal and haline contributions remains large.  
 551 Only when the temperature-controlled and salinity-controlled effects are nearly equal (i.e.,  $Tu_x =$   
 552  $\pi/4$ ), the buoyancy gradient approaches zero, leading to the disappearance of density fronts (Figure  
 553 8).

554 The “FTLE + Temp” category is concentrated in major frontal systems, including the GS,  
 555 BMC, Benguela Current, and subpolar regions. The local-overlap patterns are consistent with the  
 556 occurrence distributions (Figure 5). Compared with  $LVI_{\text{salt}}$ ,  $LVI_{\text{temp}}$  show higher F1-scores in these  
 557 regions (Figures 5a and 5d), where thermal stratification is the primary contributor to buoyancy  
 558 gradients (Figures 4c and 4d). At the same time, FTLE also maintains substantial overlap with  
 559 density fronts in these regions (Figures 5c and 5f), where density fronts are controlled jointly by  
 560 dynamic deformation and thermal stratification. In winter, the “FTLE + Temp” category becomes  
 561 more widespread because surface cooling steepens thermal gradients, while stronger strain and  
 562 frontogenesis enhance tracer convergence and sharpens frontal structures (Brannigan et al., 2015;  
 563 Prend et al., 2025).

564 The “FTLE + Salt” category is most prominent in regions influenced by freshwater forcing,  
 565 including the tropical Atlantic influenced by the Amazon and Congo river plumes, as well as the  
 566 boundary currents in the subpolar North Atlantic (Figures 4c and 4d). Meanwhile, FTLE and  
 567  $LVI_{\text{salt}}$  have higher F1-scores than  $LVI_{\text{temp}}$  (Figure 5). In these areas, freshwater inputs from rivers,  
 568 sea-ice melt, and precipitation generate strong salinity gradients intensified by deformation of the  
 569 velocity field.

570



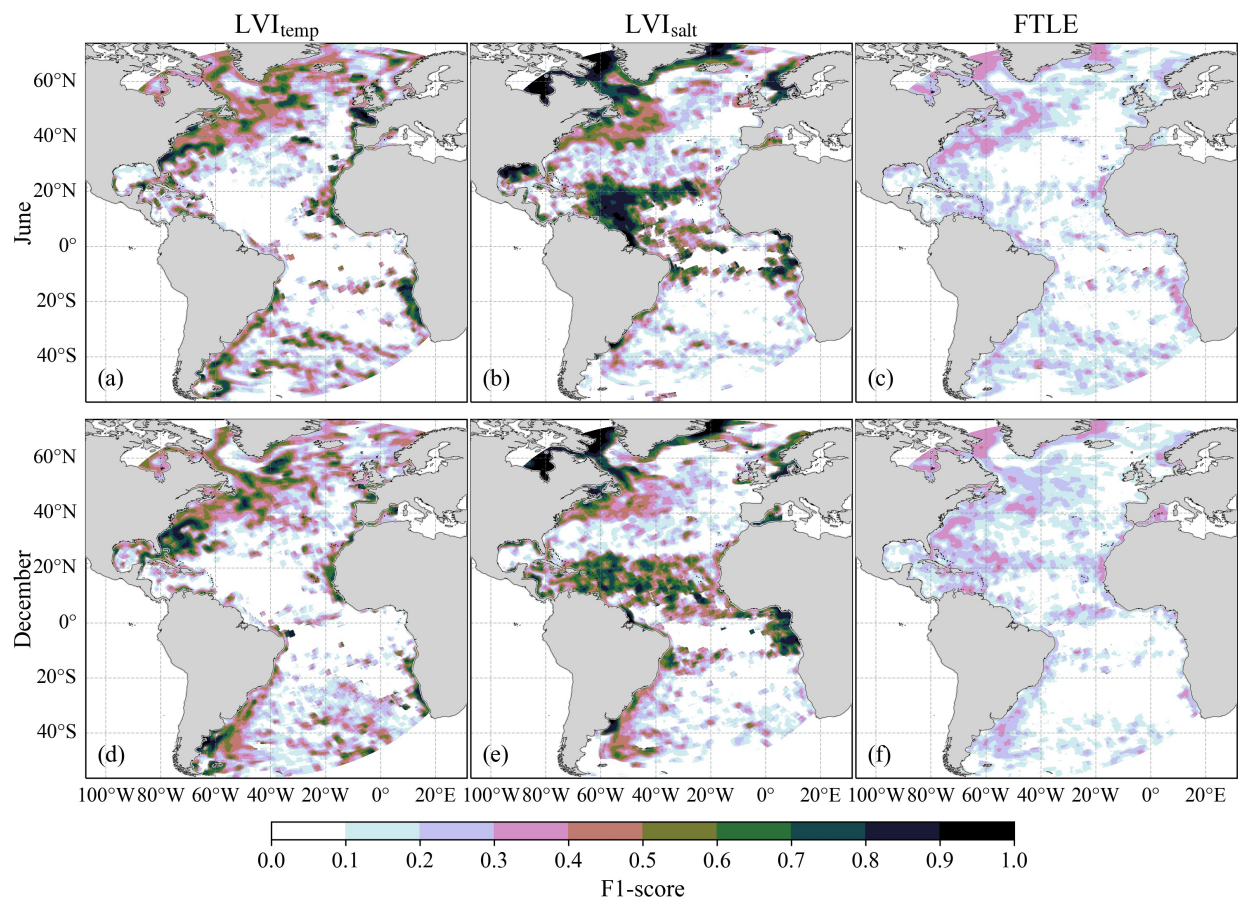
571

572 Figure 4. Dominant co-occurrence types of dynamical (FTLE), thermal ( $LVI_{\text{temp}}$ ), and haline fronts  
 573 ( $LVI_{\text{salt}}$ ) with density fronts ( $LVI_{\text{buoy}}$ ) in June and December. (a, b) show regions where all four

574 types of fronts are active simultaneously (“All”). (c, d) show two-front combinations with density  
 575 fronts (“Temp + Salt”, “FTLE + Temp”, and “FTLE + Salt”). (e, f) show regions where only one  
 576 type co-occurs with density fronts (“Temp”, “Salt”, and “FTLE”). Donut charts summarize the  
 577 percentage distribution of  $Tu_x$  categories for each dominant frontal type co-occurring with density  
 578 fronts in summer and winter. The percentage beside each “donut” indicates the area fraction of the  
 579 co-occurrence category relative to all density-front occurrences within the corresponding season.  
 580 Summer is defined as June in the Northern Hemisphere and December in the Southern Hemisphere,  
 581 and vice versa for winter.

582 The categorical map of Turner angle in Figure S21 is consistent with the annual-mean  
 583 large-scale surface Turner angle shown in Figure 3d of Whalen and Drushka (2025), who used the  
 584 monthly World Ocean Atlas sea surface temperature and salinity ( $1^\circ$  resolution). Both studies  
 585 show salinity-dominant fronts in the Tropical Atlantic (due to the influence of major river plumes)  
 586 and temperature-dominant fronts in the subtropical Atlantic. Additionally, we find more salinity-  
 587 controlled compensation in the subtropical Atlantic, a process known to occur at submesoscale  
 588 (Coadou-Chaventon et al., 2024; Spiro Jaeger & Mahadevan, 2018).

589



590  
 591 Figure 5. Detection performance of surface submesoscale density fronts in the Atlantic Ocean. (a–  
 592 c) show the F1-scores of LVI<sub>temp</sub>, LVI<sub>salt</sub>, and FTLE relative to density fronts in June. (d–f) show  
 593 the same for December.

### 594 4.3 Complementary value of FTLE and LVI

595 The comparison among  $LVI_{temp}$ ,  $LVI_{salt}$ , and FTLE shows that no single diagnostic  
596 provides a complete representation of density fronts across the Atlantic (Figures 4e and 4f). Tracer-  
597 based LVI is effective when the selected tracer dominates density gradients, but becomes less  
598 reliable when density variability is controlled by the other tracer. FTLE reduces this tracer-specific  
599 bias. In salinity-dominated regions such as the Amazon plume and the Gulf of Guinea,  $LVI_{temp}$   
600 fails to capture density fronts, with F1-scores approaching zero (Figures 5a and 5d). FTLE still  
601 achieves moderate F1-scores (Figures 5c and 5f). Similarly, in temperature-dominated regions  
602 such as the BMC, Benguela Current, Bay of Biscay, and the subtropical South Atlantic, FTLE  
603 clearly outperforms  $LVI_{salt}$ . These regional contrasts indicate that FTLE is less constrained by  
604 whether density variability is primarily controlled by temperature or salinity.

605 The regional examples clarify how FTLE complements tracer-based LVI. In the first case,  
606 temperature and salinity gradients are weak but reinforce each other, enhancing the density  
607 gradient (Figures 7a–e). As a result, density fronts are detected despite the absence of temperature  
608 or salinity fronts and align well with FTLE structures. In the second case, strong temperature and  
609 salinity fronts are observed; however, their compensating effect weakens the net buoyancy  
610 gradient, leading to sparse density fronts (Figures 7f–j). Although both density and dynamical  
611 fronts are limited in number, they exhibit greater spatial overlap because they co-locate near major  
612 frontal zones. These cases demonstrate FTLE’s ability to detect density fronts even in the absence  
613 of strong thermohaline gradients, and its insensitivity to thermohaline compensation that may  
614 obscure density fronts in Eulerian diagnostics.

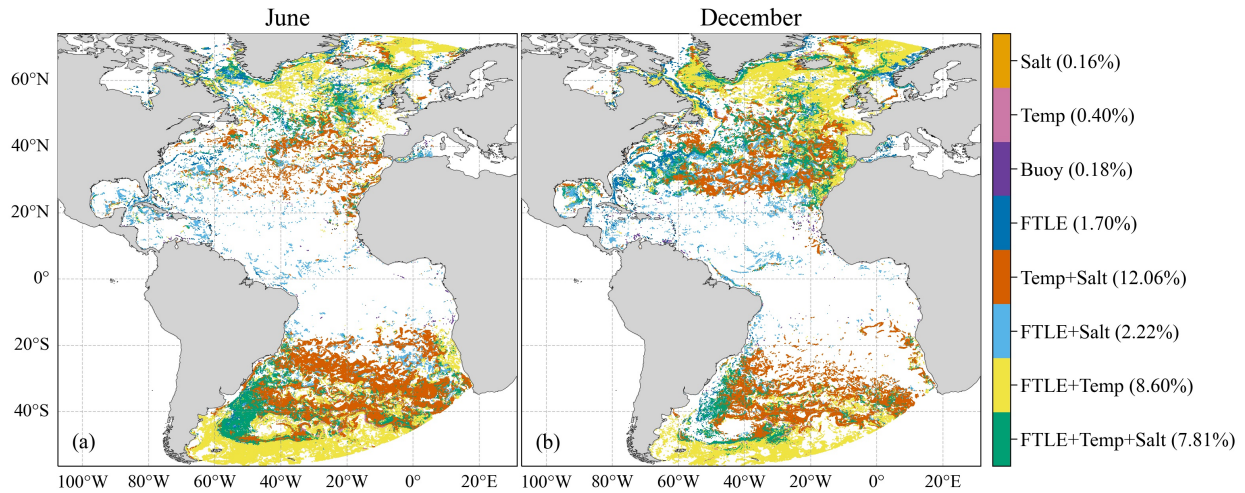
615 The complementary value of FTLE and LVI is further supported by the joint probability  
616 density distribution of front intensity and  $Tu_x$  (Figure 8). The probability density distributions of  
617  $LVI_{temp}$  and  $LVI_{salt}$  are concentrated under temperature- and salinity-controlled conditions,  
618 respectively (Figures 8c and 8d). In contrast, FTLE is distributed more broadly across different  
619 thermohaline regimes (Figure 7a).  $LVI_{temp}$  and  $LVI_{salt}$  show a tendency toward the compensation  
620 side, whereas  $LVI_{buoy}$  is more strongly skewed toward the reinforcing side (Figure 8b).  
621 Furthermore, both FTLE and  $LVI_{buoy}$  achieve the highest probabilities at  $Tu_x = \pm\pi/2$ , corresponding  
622 to strong temperature gradients, and at  $Tu_x = 0$ , corresponding to strong salinity gradients.

623 The above findings reflect the complementary value of the Lagrangian FTLE flow, which  
624 integrates deformation that tends to sharpen existing buoyancy gradients regardless of whether  
625 they are controlled primarily by temperature or salinity. Therefore, combining FTLE with Eulerian  
626 LVIs can provide more complete and less biased detection of density fronts across contrasting  
627 regimes.

### 628 4.4 Limitations of front diagnostics

629 Even with multiple diagnostics, density-front identification requires caution under  
630 thermohaline compensation. Near  $Tu_x = \pi/4$ , the thermal and haline contributions to buoyancy  
631 almost cancel each other out (Figure 8b).  $LVI_{buoy}$  shows a minimum, while the probability densities  
632 of  $LVI_{temp}$  and  $LVI_{salt}$  remain relatively large. FTLE also exhibits high probabilities of occurrence  
633 due to its insensitivity to thermohaline compensation. For example, in Figures 7k–o, density fronts  
634 remain sparse because temperature and salinity gradients compensate each other. However, strong  
635 velocity shear around eddy boundaries sustains intense stretching, giving rise to prominent FTLE  
636 structures.

637 In addition to the front types that co-occur with density fronts, large parts of the Atlantic  
 638 are also dominated by co-occurrence types without density fronts (Figure 6). The dominant  
 639 categories are “Temp + Salt”, “FTLE + Temp”, and “FTLE + Temp + Salt”, accounting for 12.06%,  
 640 8.60%, and 7.81% of all classified categories area, respectively. Their broad distributions,  
 641 particularly in the subpolar North Atlantic and the South Atlantic, indicate that tracer gradients  
 642 and deformation can coexist even when the instantaneous density-gradient signal is weak or  
 643 compensated. These results demonstrate that tracer-based or velocity-based diagnostics may over-  
 644 identify density fronts when used independently, especially under thermohaline compensation.



645  
 646 Figure 6. Dominant front types and co-occurrence types without density fronts. “FTLE + Temp +  
 647 Salt”, “FTLE + Temp”, “FTLE + Salt”, and “Temp + Salt” indicate the simultaneous occurrence  
 648 of the corresponding front types. “FTLE”, “Buoy”, “Temp”, and “Salt” refer to the most active  
 649 front types. Percentages for each co-occurrence category are shown in the corresponding panels.

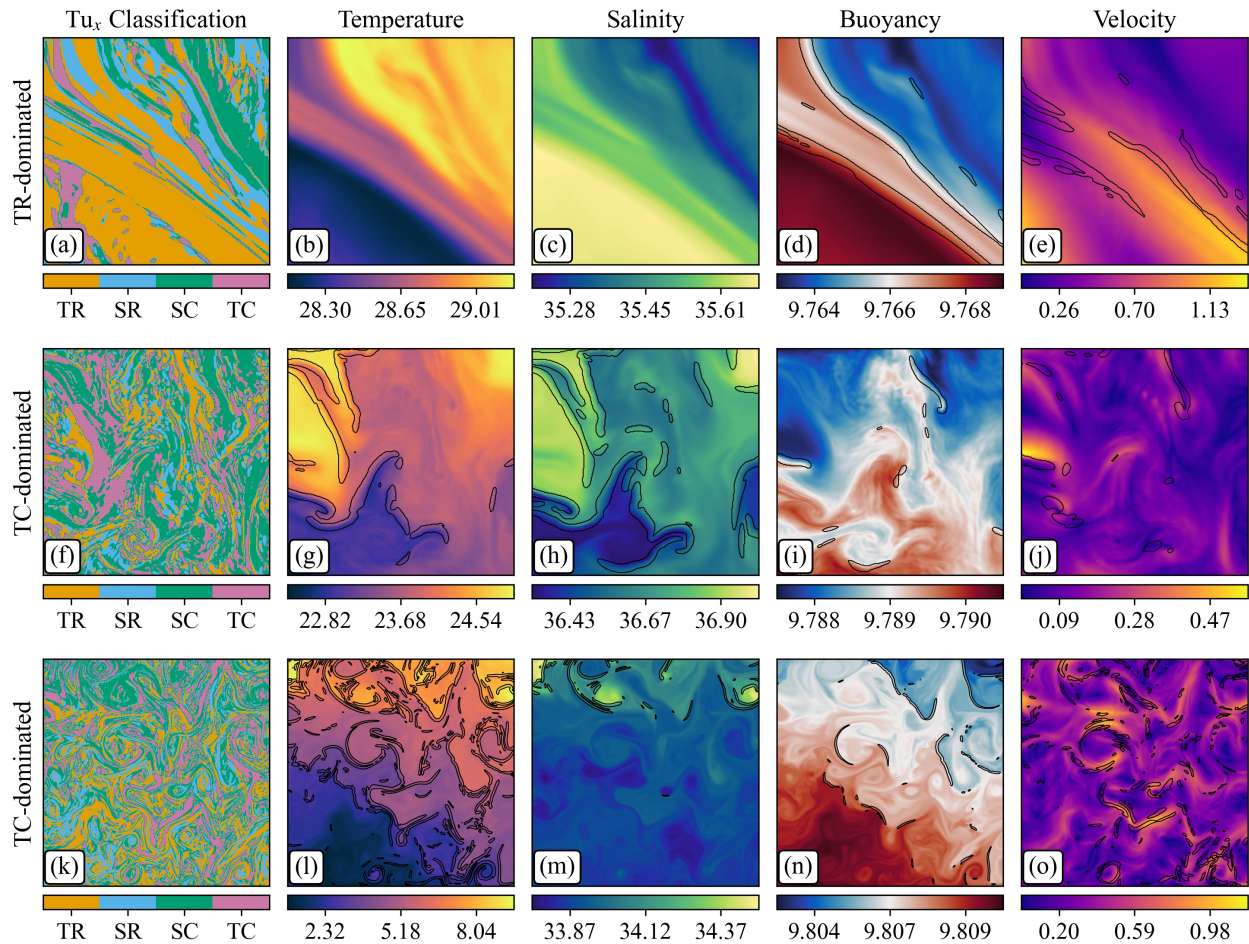
650 Spatial offset is another limitation of FTLE on density front detection. FTLE integrates  
 651 deformation along trajectories over a finite interval, whereas  $LVI_{buoy}$  is an instantaneous Eulerian  
 652 measure. As a result, FTLE can highlight regions of accumulated stretching that do not necessarily  
 653 coincide with the instantaneous maxima of buoyancy gradients, leading to spatial offsets between  
 654 dynamical and density fronts (Siegelman et al., 2020). Eddy peripheries may also generate  
 655 stretching and filamentation while vertical relative vorticity or buoyancy gradients remain weak  
 656 (Shcherbina et al., 2013).

657 Mismatch can also arise from the way frontal signals are extracted. In our detection  
 658 framework, submesoscale fronts are identified when the LVI and FTLE fields exceed the 90th-  
 659 percentile threshold. Therefore, density fronts can remain active due to strong Eulerian tracer  
 660 gradients even when Lagrangian stretching does not generate an FTLE signal exceeding the  
 661 threshold used to identify dynamical fronts. These limitations explain why FTLE has lower F1-  
 662 scores relative to density fronts (Figure 5).

663 Overall, these results demonstrate that submesoscale density fronts in the Atlantic Ocean  
 664 arise from the combined effects of Lagrangian dynamical deformation and Eulerian thermohaline  
 665 gradients. The ability of  $LVI_{temp}$  and  $LVI_{salt}$  to detect submesoscale density fronts is limited by  
 666 whether temperature or salinity dominates the local buoyancy gradient. As a result, their  
 667 performance varies substantially across regions and seasons. In contrast, FTLE is less sensitive to  
 668 thermohaline effects and provides a more robust detection capability for submesoscale density

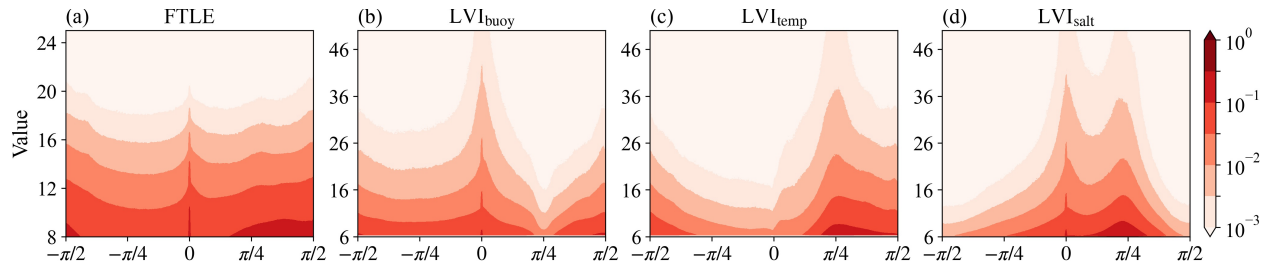
669 fronts across contrasting thermohaline regimes. It is particularly valuable in regions where either  
 670 temperature or salinity alone provides an incomplete representation of density variability.  
 671 However, when temperature–salinity compensation severely suppresses density gradients,  $LVI_{temp}$ ,  
 672  $LVI_{salt}$ , and FTLE may all show reduced correspondence with density fronts. In general, dynamical  
 673 and density fronts reflect complementary aspects of frontogenesis, linking the sharpening of lateral  
 674 density gradients to deformation in the velocity field. Therefore, they are expected to be strongly  
 675 related, even if they do not perfectly overlap spatially. From a practical detection perspective, the  
 676 combined use of FTLE and LVI-based methods provides a more comprehensive framework for  
 677 distinguishing submesoscale density fronts across the Atlantic Ocean.

678



679

680 Figure 7. Spatial distribution of  $Tu_x$  classifications and corresponding physical fields with frontal  
 681 indicators. (a), (f), and (k) represent regions of TR, TC, and TC. (b–e), (g–j), and (l–o) present  
 682 temperature, salinity, buoyancy, and velocity for the same regions, respectively, with contours  
 683 indicating thermal, haline, density, and dynamical fronts.



684

685 Figure 8. Joint probability density distributions of front intensity and  $Tu_x$  for (a) dynamical fronts,  
 686 (b) density fronts, (c) thermal fronts, and (d) haline fronts. The x-axis represents  $Tu_x$  ranging from  
 687  $-\pi/2$  to  $\pi/2$ , and the y-axis shows the corresponding frontal intensity. Color shading indicates the  
 688 probability density on a logarithmic scale.

## 689 **5 Performance of FTLE derived from the coarse velocity**

690 The above results show that FTLE computed from hourly 1-km velocity can robustly detect  
691 density fronts across contrasting thermohaline regimes and is less affected by temperature–salinity  
692 compensation than the LVI method. These findings raise a critical question: to what extent can  
693 FTLE retain this performance when applied to the much coarser velocity fields available from  
694 present-day satellite observations? To address this, we evaluate FTLE computed from coarser  
695 geostrophic velocities representative of satellite altimetry. We apply a uniform  $12.5 \times 12.5$  km  
696 spatial filter to daily geostrophic velocity fields, matching the resolution used by AVISO FSLE  
697 product. Previous studies show that the effective spatial resolution of AVISO velocity fields can  
698 exceed 100 km, due to measurement noise and along-track sampling constraints (Ballarotta et al.,  
699 2019; Picard et al., 2025). Therefore, we additionally compute FTLE from velocity fields  
700 smoothed to a 100-km resolution, representing the lower bound of altimetric resolving capability.

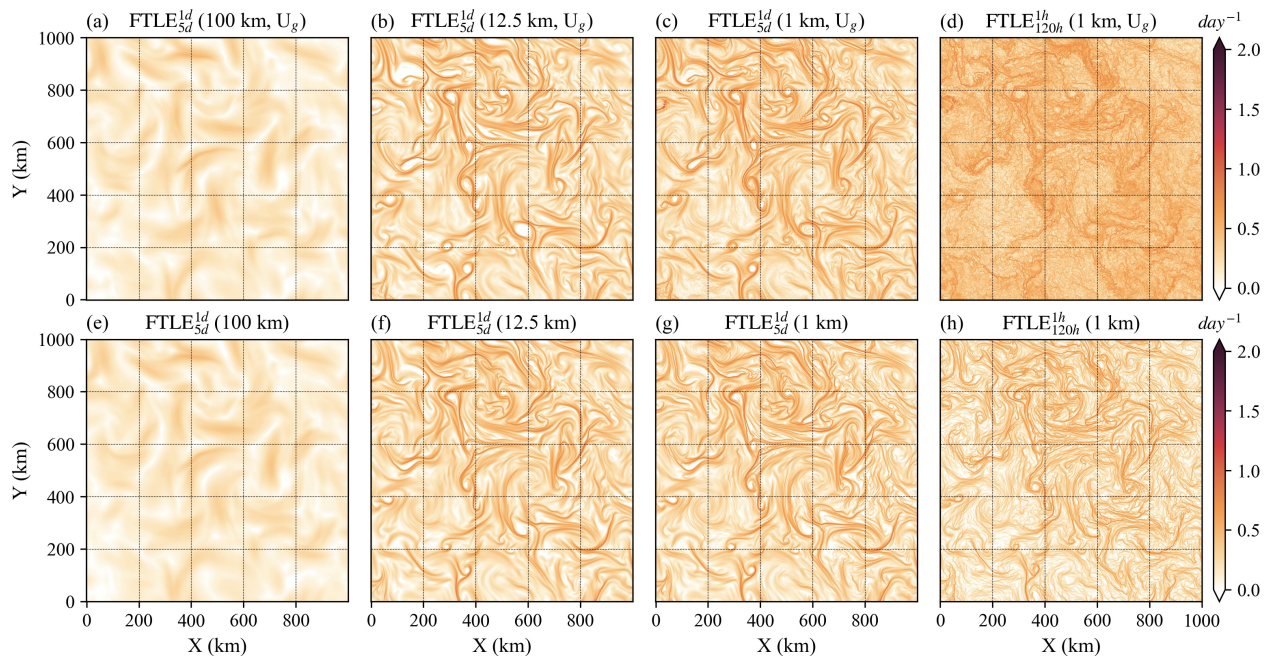
701 All FTLE computations use  $\delta_0 = 1$  km,  $T = 5$  days, and  $dt = 0.5$  days. For the 12.5-km  
702 geostrophic velocity case (Figure S22), choosing  $\delta_0 = 1$  km preserves mesoscale structures while  
703 still revealing fine-scale filaments. These filaments are not numerical artefacts, as their  
704 morphological coherence and spatial alignment closely match those obtained from daily 1-km  
705 velocities (Figures 9b and 9c). This consistency across resolutions provides confidence that FTLE  
706 retains meaningful information even when the underlying flow is coarsened.

707 FTLE is highly sensitive to the integration time  $T$  (Figures S22 d–f and S23 d–f). If  $T$  is  
708 too short, deformation structures remain underdeveloped; if too long, short-lived fronts are missed  
709 (Prants et al., 2014). Fahlbusch et al. (2024) showed that near-surface density variability is  
710 dominated by processes on time scales up to  $\sim 10$  days, so we evaluate FTLE over this range and  
711 calculate the F1 score between FTLE-derived fronts and LVI buoy fronts (Figure S24). For the  
712 12.5-km velocity fields, the F1 score peaks at  $T = 3$  days and then decreases steadily as  $T$  increases,  
713 indicating that excessive integration tends to smear or displace fine-scale fronts. For the 100-km  
714 velocity fields, the F1 score increases gradually and reaches a maximum at  $T = 7$  days, reflecting  
715 the slower evolution of fronts at coarser resolution. Considering both resolutions, an integration  
716 time of 5 days provides a balanced performance, maintaining relatively high F1 scores for both  
717 cases. Additionally, we use  $dt = 0.5$  day, which avoids numerical artefacts arising from coarse  
718 steps (e.g.,  $dt = 1$  day; Figure S22i) while remaining computationally efficient. We do not smooth  
719 the FTLE fields using an  $11 \times 11$  mean filter because the underlying velocity field already has a  
720 coarse resolution, and with a 5-day integration the FTLE derived from daily 12.5-km or 100-km  
721 velocities does not resolve the same fine-scale structures produced by FTLE computed from hourly,  
722 1-km velocities. Instead, we apply the same masking strategy to the FTLE fields to extract the  
723 dynamical fronts, which is described in Section 2.4.

724 As expected, increasing the temporal and spatial resolution of the velocity field  
725 progressively reveals sharper and more filamentary LCS signatures (Figures 9a–9d). As  
726 geostrophic velocity is divergence-free, FTLE cannot directly identify any Eulerian convergence  
727 zones. However, by comparing with FTLE derived from the full velocity smoothed to the same  
728 resolution, we find a high similarity with FTLE derived from geostrophic velocity (Figures 9e–  
729 9h). Moreover, as the velocity field becomes coarser, the structures captured by the two FTLE  
730 become more identical (Figures 9a–9c and 9e–9g). These findings imply that in nondivergent  
731 flows (geostrophic velocity or sufficiently coarse velocity), frontogenesis is associated with  
732 kinematic criteria other than divergence. As argued by Hetland et al. (2025), when the flow is  
733 primarily nondivergent ( $\delta \approx 0$ ), frontal maintenance can be dominated by the effective (normal)

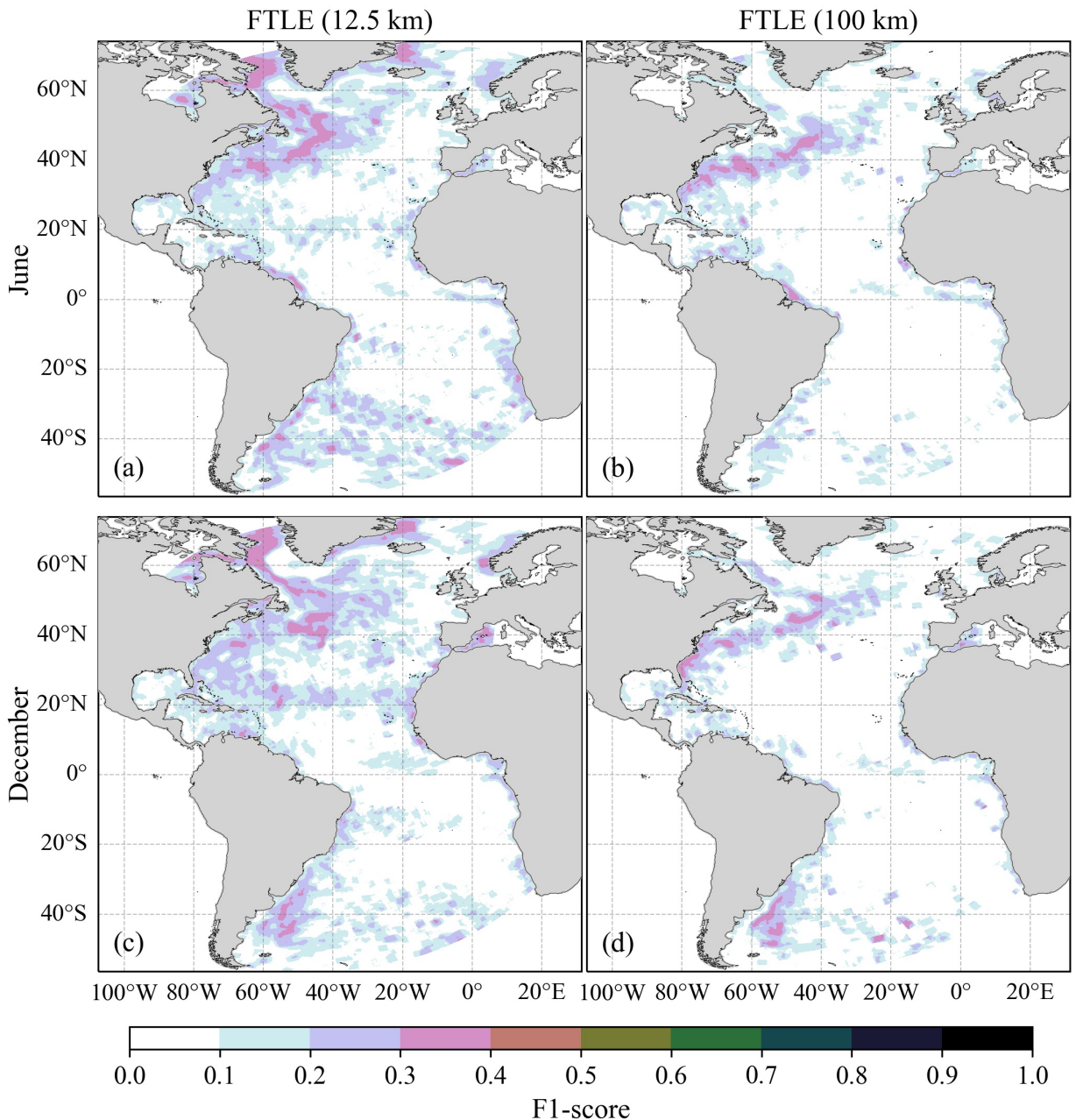
734 strain in a frontal coordinate system. In that case, fronts strengthen through cross-front contraction  
735 and along-front divergence ( $\tilde{u}_x \approx -\tilde{v}_y$ ), also called confluent contraction (see Figure 10 in Hetland  
736 et al., 2025). Such kinematics organize coherent attracting/stretching LCS, yielding FTLE ridges  
737 aligned with fronts even in the absence of explicit horizontal convergence. Conversely, as the  
738 fields are resolved at 1-km and hourly resolution (Figures 9d and 9h), the role of divergence in  
739 frontogenesis becomes increasingly important. Deformation in the hourly, 1-km geostrophic flow  
740 is strongly modulated by high-frequency oscillations, making it difficult for a coherent,  
741 cumulatively effective net contraction to persist over the finite integration period and thereby  
742 smearing attracting FTLE ridges. By contrast, the full flow includes ageostrophic cross-front  
743 circulations that provide a more sustained net contraction, making the LCS structures in Figure 9h  
744 noticeably sharper and more coherent. More generally, the submesoscale full flow permits  
745 substantial horizontal convergence, allowing FTLE to capture more explicit convergence  
746 signatures. Overall, these results indicate that FTLE can diagnose frontal LCS through strain-  
747 dominated frontogenesis in nondivergent geostrophic flows.

748 Consistent with the structural refinement, the mean F1-scores increase from 0.13 for the  
749 100-km full velocity field to 0.18 for the 12.5-km field and 0.21 for the 1-km field. The small  
750 difference between the 1-km and 12.5-km results indicates that FTLE retains useful diagnostic  
751 skill for detecting submesoscale density fronts even when derived from coarse-resolution velocity.  
752 For geostrophic flow, the mean F1-scores of 100-km and 12.5-km fields are 0.13 and 0.16,  
753 respectively. At the 12.5-km scale, the skill of FTLE in identifying density fronts is partly limited  
754 when computed from the geostrophic flow, because the geostrophic component lacks convergence  
755 that sharpens density fronts. In contrast, at the 100-km scale both the full and geostrophic flows  
756 are nearly non-divergent and dominated by large-scale balanced dynamics, leading to similarly  
757 low F1 scores. At this extreme level of smoothing, only dynamically energetic regions retain  
758 detectible FTLE ridges, such as western boundary currents, equatorial jets, and the South Atlantic  
759 westerly wind belt (Figures 10b and 10d). This finding reflects the fact that strong background  
760 shear and strain in these regions can survive heavy smoothing, whereas weaker submesoscale  
761 features cannot.



763

764 Figure 9. FTLE computed in a North Atlantic subregion using geostrophic (top row) and  
 765 (bottom row) velocity fields at different spatial and temporal resolutions. (a) FTLE derived from  
 766 daily geostrophic velocity fields that were uniformly filtered to  $100 \text{ km} \times 100 \text{ km}$  ( $T = 5$  days,  $dt$   
 767  $= 0.5$  days). (b) FTLE derived from daily geostrophic velocity fields that were uniformly filtered  
 768 to  $12.5 \text{ km} \times 12.5 \text{ km}$  ( $T = 5$  days,  $dt = 0.5$  days). (c) FTLE derived from daily 1-km geostrophic  
 769 velocity fields ( $T = 5$  days,  $dt = 0.5$  days). (d) FTLE derived from hourly 1-km geostrophic  
 770 velocity fields ( $T = 120\text{h}$ ,  $dt = 12\text{h}$ ). (e–h) same as (a–d) but using full velocity fields. All FTLE fields use  
 771 an initial separation  $\delta_0 = 1 \text{ km}$ .



773

774 Figure 10. FTLE detection performance based on geostrophic velocities at different resolutions in  
 775 June and December. The F1-scores shown here are computed between density fronts ( $LVI_{buoy}$ ) and  
 776 dynamical fronts (FTLE). (a, c) F1-scores for FTLE computed from the daily 12.5-km velocity  
 777 fields. (b, d) F1-scores for FTLE computed from the daily 100-km velocity fields.

## 778 6 Conclusions

779 In this study, we use a high-resolution, submesoscale permitting simulation of the Atlantic  
 780 Ocean (GIGATL1) to investigate the detection and dynamics of surface submesoscale fronts. We  
 781 apply two complementary diagnostics: the Eulerian LVI, which captures thermal, haline, and

782 density fronts based on the variability of surface tracers, and the Lagrangian FTLE, which  
783 identifies dynamically active regions associated with stretching and tracer convergence. By  
784 implementing scale-selective temporal and spatial filtering, and a consistent thresholding approach,  
785 we extract frontal features at scales of 1–10 km and 1–10 days, and compare their dynamical  
786 characteristic and spatial distributions. We further examine how dynamical, thermal, and haline  
787 fronts relate to density fronts and assess the potential of FTLE and temperature-based LVI as  
788 indicators for density-front detection.

789 The detected dynamical and density fronts exhibit the strongest submesoscale signatures,  
790 indicating their closer connection to deformation-driven frontogenesis and ageostrophic secondary  
791 circulation. Submesoscale fronts exhibit a clear asymmetry toward cyclonic vorticity and  
792 convergence, resulting from the asymmetric response of the vorticity tendency to vortex stretching.  
793 These findings are consistent with previous studies and support the validity of our submesoscale  
794 front detection approach.

795 Spatial and seasonal patterns of frontal activity across the Atlantic indicate that dynamical  
796 fronts are both widespread and persistent. Their frequency increases during winter, primarily due  
797 to intensified frontal dynamics, including elevated strain rate and vorticity. Thermohaline fronts  
798 exhibit greater regional and seasonal variability. Thermal fronts are more active in winter as a  
799 result of surface cooling, whereas haline fronts reach peak activity in summer near major river  
800 outflows, including the Amazon and Congo. Density fronts, influenced by either temperature- or  
801 salinity-controlled effects, exhibit a broader distribution and combine the characteristics of both  
802 thermal and haline fronts. In tropical and subtropical gyres, thermohaline compensation frequently  
803 reduces density gradients. This process leads to sparse density fronts, even when dynamical  
804 activity persists.

805 The co-occurrence of density fronts with FTLE and at least one tracer-based front indicates  
806 that deformation-driven frontogenesis commonly sharpens pre-existing thermal or haline gradients,  
807 thereby producing density fronts with both dynamical and thermohaline signatures. The  
808 comparison among  $LVI_{temp}$ ,  $LVI_{salt}$ , and FTLE reveals their complementary roles in density-front  
809 detection.  $LVI_{temp}$  and  $LVI_{salt}$  are effective when density variability is dominated by temperature  
810 or salinity, respectively, but they may become less reliable when the other tracer controls the  
811 buoyancy gradient. In compensated regions,  $LVI_{temp}$  and  $LVI_{salt}$ -based fronts may produce false  
812 positives, whereas in reinforced regions, they may miss density fronts. In contrast, FTLE is less  
813 sensitive to thermohaline effects, allowing it to identify density fronts across contrasting  
814 thermohaline regimes. Nevertheless, FTLE can identify stretching structures that do not coincide  
815 with instantaneous buoyancy gradients, and may also cause false detection of density fronts when  
816 thermohaline compensation suppresses the density signal.

817 The methodology developed here can be applied to observational datasets, especially those  
818 combining high-resolution satellite-derived SST and surface velocities. Unfortunately, satellite  
819 SSS observations do not resolve submesoscale features (Boutin et al., 2018), but FTLE may  
820 partially compensate that and provide information about density fronts in salinity dominated  
821 regions. However, care must be taken when interpreting FTLE structures without direct density  
822 observations, as tracer compensation may obscure density fronts. In addition, our results indicate  
823 that while FTLE retains meaningful skill at mesoscale-resolving resolutions (e.g., 12.5 km), its  
824 performance diminishes at altimetry scales approaching ~100 km, where only the strongest  
825 deformation features remain detectable. This behavior reflects the dependence of FTLE on the  
826 resolved strain–rate variability, emphasizing the importance of velocity resolution when applying

827 FTLE to observational products. As higher-resolution surface velocity products from the Surface  
828 Water and Ocean Topography (SWOT) mission become available, FTLE-based diagnostics are  
829 expected to better resolve submesoscale frontal structures. Future work should consider integrating  
830 additional satellite variables such as ocean color, and using machine learning to classify frontal  
831 zones based on a combination of the different diagnostics presented here, as well as information  
832 on the vertical density profiles (Yao et al., 2025; Yao & Taylor, 2026). Further improvements may  
833 include adaptive thresholding for different dynamic regions and incorporating vertical data from  
834 model reanalysis or autonomous observations. Overall, this study establishes a benchmark  
835 framework for interpreting submesoscale frontal dynamics and guides the design of observational  
836 strategies to resolve fine-scale ocean processes.

### 837 **CRedit authorship contribution statement**

838 **Qian Liu:** Conceptualization, Methodology, Software, Validation, Formal analysis, Data  
839 Curation, Writing – Original Draft, Writing – Review & Editing, Visualization. **Dante C.**  
840 **Napolitano:** Investigation, Methodology, Writing – Original Draft, Writing – Review & Editing,  
841 Visualization, Supervision. **Jonathan Gula:** Investigation, Methodology, Conceptualization,  
842 Resources, Data Curation, Writing – Original Draft, Writing – Review & Editing, Supervision,  
843 Project administration. **Mengjiao Wang:** Writing – Review & Editing. **Yingjie Liu:** Writing –  
844 Review & Editing. **Le Gao:** Writing – Review & Editing. **Xiaofeng Li:** Conceptualization,  
845 Resources, Writing – Review & Editing, Supervision, Project administration, Funding acquisition.

### 846 **Declaration of competing interest**

847 The authors declare that they have no known competing financial interests or personal  
848 relationships that could have appeared to influence the work reported in this paper.

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### 860 **Data availability**

861 The information about the GIGATL1 can be found at <https://doi.org/10.5281/zenodo.4948523>.  
862 AVISO FSLE product can be found at: <https://doi.org/10.24400/527896/A01-2022.002>.

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